Al-Balqa' Applied University

Al-Huson University College

Mechanical Engineering Department

Mechanical design (30131326)

Professor Dr. Musa K. AlAjlouni

Second Semester 2014/2015

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Mechanical design (30131326)

Course Title:	Mechanical design (30131326)
Prerequisite:	Strength of Material (30129212)
Text Book:	Shigley's Mechanical Engineering Design, **Budynas–Nisbett**, 2011, Ninth edition, McGraw-Hill Book Company..
Providing Dept.:	Mechanical Engineering
Instructor:	Prof. Dr. Musa K. AlAjlouni **Office No:** **Tel**:
Level:	3rd year **Credit Hours**: 3
Semester:	Second 2014-2015
Time:	(08:00-09:00) Sundays, Tuesdays and Thursdays.
Office Hours:	(to be announced soon after).

Time Schedule:

Duration: 16 Weeks   **Lectures:** 3 hours / week

Objectives: Mechanical Engineering design is a study of design-making processes. It utilizes mathematics, the material sciences and the engineering-mechanics sciences. In this course concentration will be on machine elements design rather than machine design. This course will cover the following topics:

Course Contents

Subject	week	Subject	week
1. Review of machine Drawing includes:	1-4	**First Exam** 2nd lecture 7th week (10th of March 2015) and submission of project one (coupling);	
a. Assembly and detail drawings.			
b. Manufacturing processes			
c. Fit and Tolerance.		5. Power Screw and Screw Fasteners	8-9
d. Surface roughness.		6. Welding and riveted joints.	9
2. Introduction to design include:		**Second Exam** 2nd lecture 12th week (21st of April 2015) and submission of project two (Jack).	10-11
a. The meaning and phases of design.	5		
b. Simple stresses.		7. Clutch and brake design	13
c. Safety factors.		8. Mechanical Springs	14
d. Stress analyses		9. Pressure Vessels (Thick and thin cylinders)	15
3. Theories of failure and bearing	6		16
4. Shaft and Couplings design	7	10. Belt, Chains Gear and Rope Design.	
		Final Exam	

Mode of Assessment

1. **First exam:**	**(20%)**
2. **Second exam:**	**(20%)**
3. **Reports, H. works, and/or Projects**	**(10%)**
4. **Final exam:**	**(50%)**

References

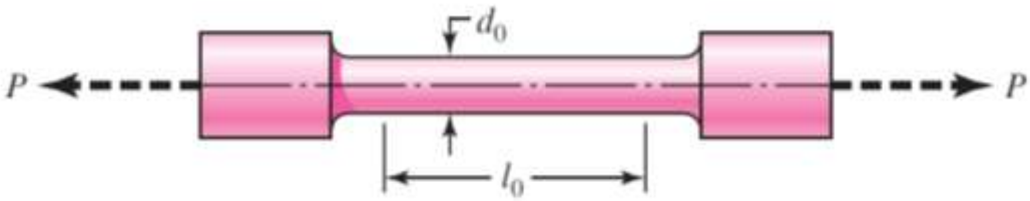
- Machine Design, Abdul Mubeen, 1995, Second edition, Khanna Publishers.
- Machine Design: Theory and Practice, A. D. Deutschman, W. J. Maichels and C.E. Wilson, 1975, First edition, Macmillan Publishing Company.
- Machine Design, A .CAD approach, A. D. Demarogonas, 2001, First edition, Wiley and sons Publishing Company.

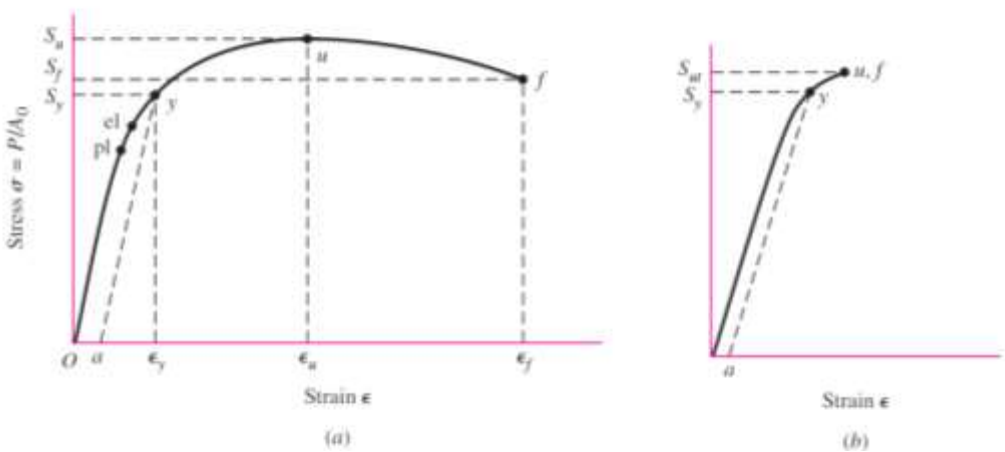
Part I

Introduction to Mechanical Engineering Design

This part includes:

- ✓ **The Nature of Mechanical Engineering Design**
- ✓ **Design phases**
- ✓ **Design Procedure**
- ✓ **Design Considerations**
- ✓ **Standards and Codes**
- ✓ **Materials Selection**
- ✓ **Dimensions, Tolerances, Limits and Fits**
- ✓ **Surface finish**
- ✓ **Relation between manufacturing processes, tolerances and surface finish**





The Nature of Mechanical Engineering Design:

Design is an iterative process with many interactive phases. Many resources exist to support the designer, including many sources of information and an abundance of computational design tools. The design engineer needs not only to develop competence in their field but must also cultivate a strong sense of responsibility and professional work ethic.

There are roles to be played by codes and standards, ever-present economics, safety, and considerations of product liability. The survival of a mechanical component is often related through stress and strength. Matters of uncertainty are ever-present in engineering design and are typically addressed by the design factor and factor of safety, either in the form of a deterministic (absolute) or statistical sense. The latter, statistical approach, deals with a design's *reliability* and requires good statistical data. In mechanical design, other considerations include dimensions and tolerances, units, and calculations.

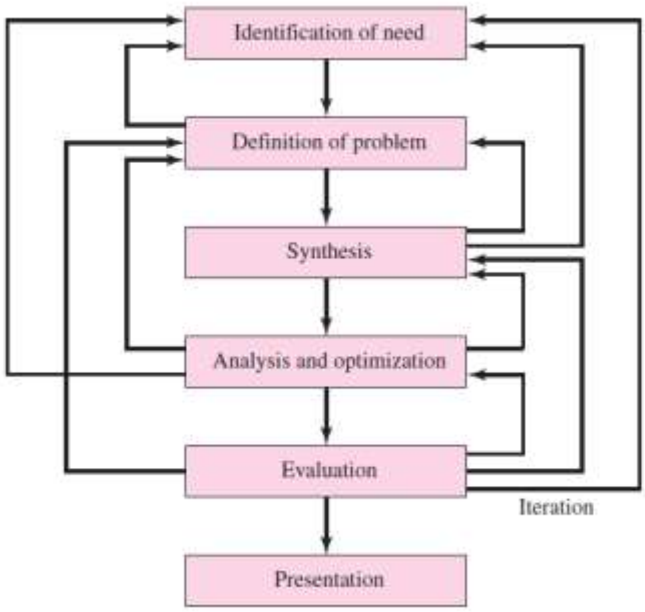
Design is an innovative and highly iterative process. It is also a decision-making process. Decisions sometimes have to be made with too little information, occasionally with just the right amount of information, or with an excess of partially contradictory information. Decisions are sometimes made tentatively, with the right reserved to adjust as more becomes known. The point is that the engineering designer has to be personally comfortable with a decision-making, problem-solving role.

Design phases:

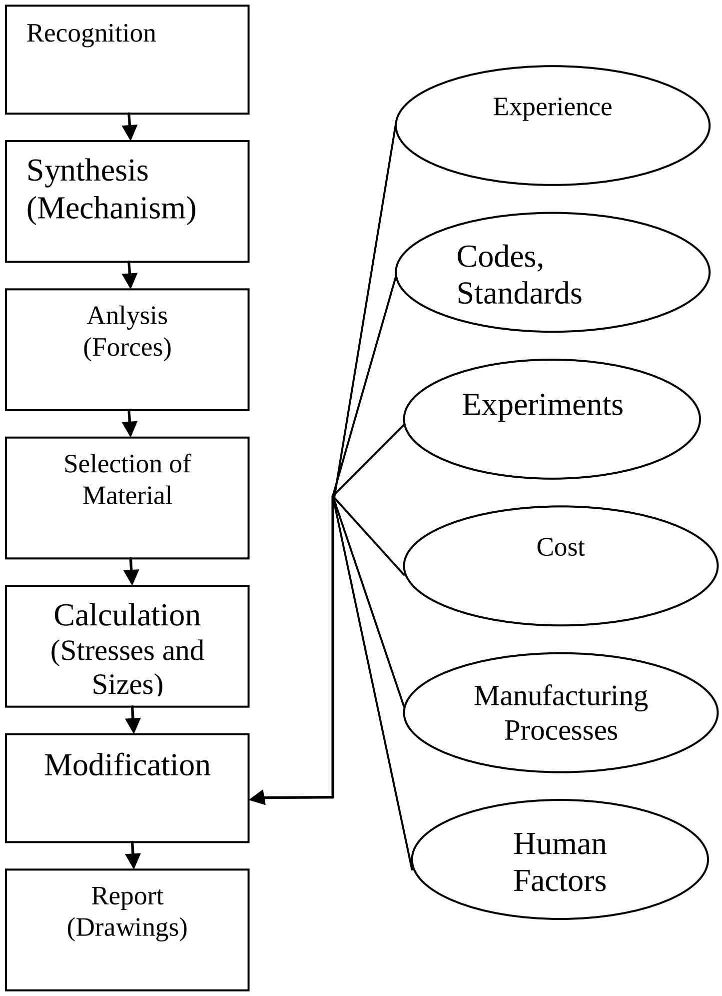
The complete design process, from start to finish, is often outlined as in Fig. 1–1. The process begins with an identification of a need and a decision to do something about it. After many iterations, the process ends with the presentation of the plans for satisfying the need. Depending on the nature of the design task, several design phases may be repeated throughout the life of the product, from inception to termination.

Figure 1–1

The phases in design, acknowledging the many feedbacks and iterations.



Design Procedure :



Design Considerations:

Sometimes the strength required of an element in a system is an important factor in the determination of the geometry and the dimensions of the element. In such a situation we say that strength is an important design consideration. When we use the expression design consideration, we are referring to some characteristic that influences the design of the element or, perhaps, the entire system. Usually quite a number of such characteristics must be considered and prioritized in a given design situation. Many of the important ones are as follows (not necessarily in order of importance):

1 Functionality
2 Strength/stress
3 Distortion/deflection/stiffness
4 Wear
5 Corrosion
6 Safety
7 Reliability
8 Manufacturability
9 Utility
10 Cost
11 Friction
12 Weight
13 Life
14 Noise
15 Styling
16 Shape
17 Size
18 Control
19 Thermal properties
20 Surface
21 Lubrication
22 Marketability
23 Maintenance
24 Volume
25 Liability
26 Remanufacturing
27 Resource recovery

Some of these characteristics have to do directly with the dimensions, the material, the processing, and the joining of the elements of the system. Several characteristics may be interrelated, which affects the configuration of the total system.

Standards and Codes:

A *standard* is a set of specifications for parts, materials, or processes intended to achieve uniformity, efficiency, and a specified quality. One of the important purposes of a standard is to place a limit on the number of items in the specifications so as to provide a reasonable inventory of tooling, sizes, shapes, and varieties.

A *code* is a set of specifications for the analysis, design, manufacture, and construction of something. The purpose of a code is to achieve a specified degree of safety, efficiency, and performance or quality. It is important to observe that safety codes *do not* imply *absolute safety*. In fact, absolute safety is impossible to obtain. Sometimes the unexpected event really does happen. Designing a building to withstand a 180 km/h wind does not mean that the designers think a 200 km/h wind is impossible; it simply means that they think it is highly improbable.

All of the organizations and societies listed below have established specifications for standards and safety or design codes. The name of the organization provides a clue to the nature of the standard or code. Some of the standards and codes, as well as addresses, can be obtained in most technical libraries. The organizations of interest to mechanical engineers are:

Aluminum Association (AA)
American Gear Manufacturers Association (AGMA)
American Institute of Steel Construction (AISC)
American Iron and Steel Institute (AISI)
American National Standards Institute (ANSI)
ASM International
American Society of Mechanical Engineers (ASME)
American Society of Testing and Materials (ASTM)
American Welding Society (AWS)
American Bearing Manufacturers Association (ABMA)
British Standards Institution (BSI)
Industrial Fasteners Institute (IFI)
Institution of Mechanical Engineers (I. Mech. E.)
International Bureau of Weights and Measures (BIPM)
International Standards Organization (ISO)
National Institute for Standards and Technology (NIST)
Society of Automotive Engineers (SAE)

Materials Selection:

The selection of a material for a machine part or structural member is one of the most important decisions the designer is called on to make. There is many important material physical properties, various characteristics of typical engineering materials, and various material production processes. The actual selection of a material for a particular design application can be an easy one, say, based on previous applications (1020 steel is always a good candidate because of its many positive attributes), or the selection process can be as involved and daunting as any design problem with the evaluation of the many material physical, economical, and processing parameters. There are systematic and optimizing approaches to material selection. Here, for simplification, we will start with steel if that’s work and more accurate method will be lifted for future. Otherwise, look at how to approach some material properties. One basic technique is to list all the important material properties associated with the design, e.g., strength, stiffness, and cost. This can be prioritized by using a weighting measure depending on what properties are more important than others. Next, for each property, list all available materials and rank them in order beginning with the best material; e.g., for strength, high-strength steel such as 4340 steel should be near the top of the list. For completeness of available materials, this might require a large source of material data. Once the lists are formed, select a manageable amount of materials from the top of each list. From each reduced list select the materials that are contained within every list for further review. The materials in the reduced lists can be graded within the list and then weighted according to the importance of each property.

Dimensions, Tolerances, Limits and Fits:

The following terms are used generally in dimensioning:

• *Basic or Nominal size.* The size we use in speaking of an element. Either the theoretical size or the actual measured size may be quite different.

• *Limits.* The stated maximum and minimum dimensions.

• *Tolerance.* The difference between the two limits.

• *Bilateral tolerance.* The variation in both directions from the basic dimension. That is, the basic size is between the two limits, for example, 1.005 ± 0.002 in. The two parts of the tolerance need not be equal.

• *Unilateral tolerance.* The basic dimension is taken as one of the limits, and variation is permitted in only one direction, for example, 1.005 +0.004 −0.000 in

• *Clearance.* A general term that refers to the mating of cylindrical parts such as a bolt and a hole. The word clearance is used only when the internal member is smaller than the external member. The *diametral clearance* is the measured difference in the two diameters. The *radial clearance* is the difference in the two radii.

• *Interference.* The opposite of clearance, for mating cylindrical parts in which the internal member is larger than the external member.

• *Allowance.* The minimum stated clearance or the maximum stated interference for mating parts. When several parts are assembled, the gap (or interference) depends on the dimensions and tolerances of the individual parts.

The designer is free to adopt any geometry of fit for shafts and holes that will ensure the intended function. There is sufficient accumulated experience with commonly recurring situations to make standards useful. There are two standards for limits and fits in the United States, one based on inch units and the other based on metric units. These differ in nomenclature, definitions, and organization. No point would be served by separately studying each of the two systems. The metric version is the newer of the two and is well organized, and so here we present only the metric version but include a set of inch conversions to enable the same system to be used with either system of units.

In using the standard, capital letters always refer to the hole; lowercase letters are used for the shaft.

The definitions illustrated in Fig. 7–20 are explained as follows:

• *Basic size* is the size to which limits or deviations are assigned and is the same for both members of the fit.

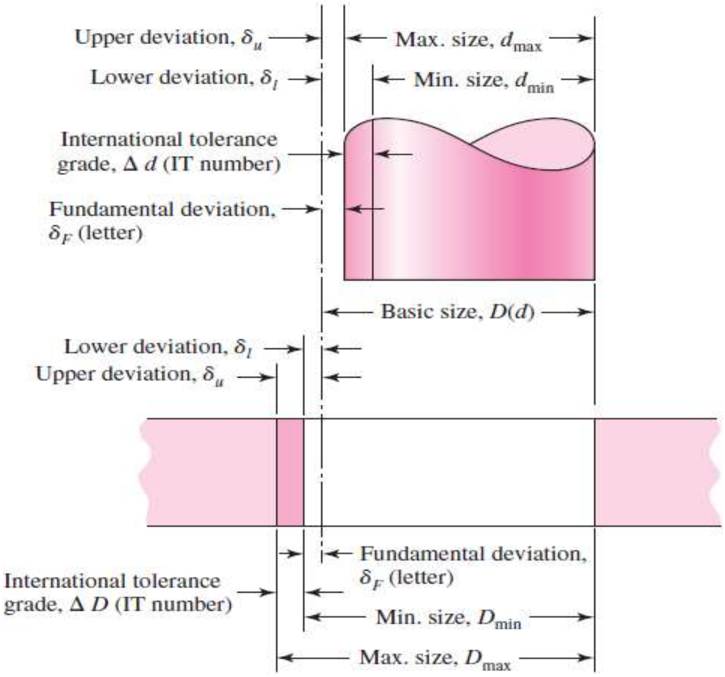
• *Deviation* is the algebraic difference between a size and the corresponding basic size.

• *Upper deviation* is the algebraic difference between the maximum limit and the corresponding basic size.

• *Lower deviation* is the algebraic difference between the minimum limit and the corresponding basic size.
• *Fundamental deviation* is either the upper or the lower deviation, depending on which is closer to the basic size.

Figure 7–20

Definitions applied to a cylindrical fit.



• *Tolerance* is the difference between the maximum and minimum size limits of a part.
• *International tolerance grade* numbers (IT) designate groups of tolerances such that the tolerances for a particular IT number have the same relative level of accuracy but vary depending on the basic size.
• *Hole basis* represents a system of fits corresponding to a basic hole size. The fundamental deviation is H.
• *Shaft basis* represents a system of fits corresponding to a basic shaft size. The fundamental deviation is h. The shaft-basis system is not included here.
• *Types of fits* are clearance, translation and interference fit.
The magnitude of the tolerance zone is the variation in part size and is the same for both the internal and the external dimensions. The tolerance zones are specified in international tolerance grade numbers, called IT numbers. The smaller grade numbers specify a smaller tolerance zone. These range from IT0 to IT16, but only grades IT6 to IT11 are needed for the preferred fits. These are listed in Tables A–11 to A–13 for basic sizes up to 16 in or 400 mm.
The standard uses *tolerance position letters*, with capital letters for internal dimensions (holes) and lowercase letters for external dimensions (shafts). As shown in Fig. 7–20, the fundamental deviation locates the tolerance zone relative to the basic size.
Table 7–9 shows how the letters are combined with the tolerance grades to establish a preferred fit. The ISO symbol for the hole for a sliding fit with a basic size of 32 mm is 32H7. Inch units are not a part of the standard. However, the designation (13.8 in) H7 includes the same information and is recommended for use here. In both cases, the capital letter H establishes the fundamental deviation and the number 7 defines a tolerance grade of IT7.

For the sliding fit, the corresponding shaft dimensions are defined by the symbol 32g6 [(13.8 in)g6]. The fundamental deviations for shafts are given in Tables A–11 and A–13. For letter codes c, d, f, g, and h,
Upper deviation = fundamental deviation
Lower deviation = upper deviation − tolerance grade
For letter codes k, n, p, s, and u, the deviations for shafts are
Lower deviation = fundamental deviation
Upper deviation = lower deviation + tolerance grade
The lower deviation H (for holes) is zero. For these, the upper deviation equals the tolerance grade.
As shown in Fig. 7–20, we use the following notation:
D = basic size of hole
d = basic size of shaft
δ_u = upper deviation
δ_l = lower deviation
δ_F = fundamental deviation
ΔD = tolerance grade for hole
Δd = tolerance grade for shaft
Note that these quantities are all deterministic. Thus, for the hole,
$D_{max} = D + \Delta D$ *and* $D_{min} = D$
For shafts with clearance fits c, d, f, g, and h,
$d_{max} = d + \delta F$ $d\text{min} = d + \delta F - \Delta d$
For shafts with interference fits k, n, p, s, and u,
$d_{min} = d + \delta_F$ $d_{max} = d + \delta_F + \Delta d$

Table of Preferred Limits and Fits for Cylindrical Parts, ANSI B4.1-1967. Preferred Metric Limits and Fits, ANSI-B4.2-1978.

Type of Fit	Description	Symbol
Clearance	*Loose running fit:* for wide commercial tolerances or allowances on external members	H11/c11
	Free running fit: not for use where accuracy is essential, but good for large temperature variations, high running speeds, or heavy journal pressures	H9/d9
	Close running fit: for running on accurate machines and for accurate location at moderate speeds and journal pressures	H8/f7
	Sliding fit: where parts are not intended to run freely, but must move and turn freely and locate accurately	H7/g6
	Locational clearance fit: provides snug fit for location of stationary parts, but can be freely assembled and disassembled	H7/h6
Transition	*Locational transition* fit for accurate location, a compromise between clearance and interference	H7/k6
	Locational transition fit for more accurate location where greater interference is permissible	H7/n6
Interference	*Locational interference fit:* for parts requiring rigidity and alignment with prime accuracy of location but without special bore pressure requirements	H7/p6
	Medium drive fit: for ordinary steel parts or shrink fits on light sections, the tightest fit usable with cast iron	H7/s6
	Force fit: suitable for parts that can be highly stressed or for shrink fits where the heavy pressing forces required are impractical	H7/u6

Table A-11

A Selection of International Tolerance Grades—Metric Series (Size Ranges Are for Over the Lower Limit and *Including* the Upper Limit. All Values Are in Millimeters)

Source: Preferred Metric Limits and Fits, ANSI B4.2-1978. See also BSI 4500.

Basic Sizes	Tolerance Grades IT6	IT7	IT8	IT9	IT10	IT11
0–3	0.006	0.010	0.014	0.025	0.040	0.060
3–6	0.008	0.012	0.018	0.030	0.048	0.075
6–10	0.009	0.015	0.022	0.036	0.058	0.090
10–18	0.011	0.018	0.027	0.043	0.070	0.110
18–30	0.013	0.021	0.033	0.052	0.084	0.130
30–50	0.016	0.025	0.039	0.062	0.100	0.160
50–80	0.019	0.030	0.046	0.074	0.120	0.190
80–120	0.022	0.035	0.054	0.087	0.140	0.220
120–180	0.025	0.040	0.063	0.100	0.160	0.250
180–250	0.029	0.046	0.072	0.115	0.185	0.290
250–315	0.032	0.052	0.081	0.130	0.210	0.320
315–400	0.036	0.057	0.089	0.140	0.230	0.360

Table A-12

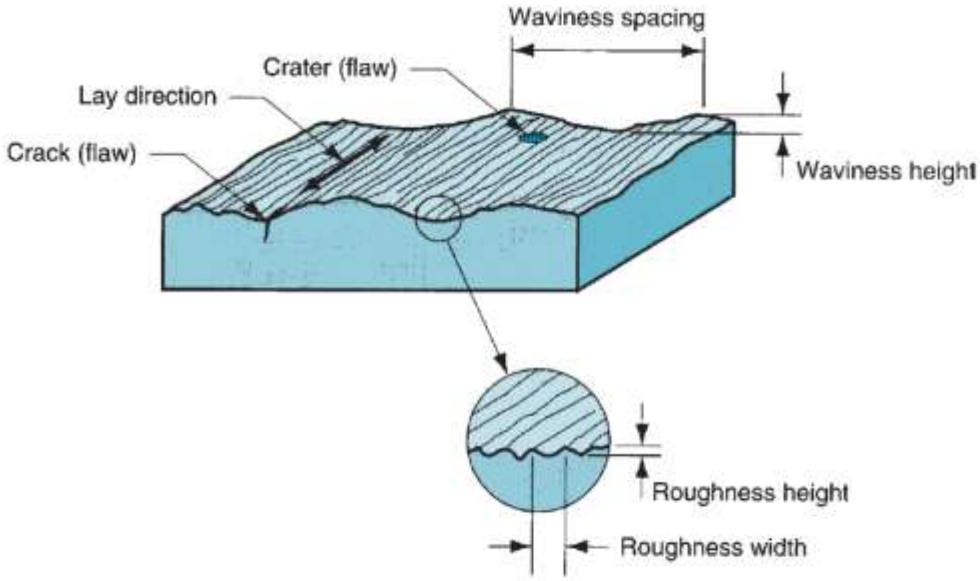
Fundamental Deviations for Shafts—Metric Series
(Size Ranges Are for *Over* the Lower Limit and *Including* the Upper Limit. All Values Are in Millimeters)
Source: Preferred Metric Limits and Fits, ANSI B4.2-1978. See also BSI 4500.

Basic Sizes	Upper-Deviation Letter c	d	f	g	h	Lower-Deviation Letter k	n	p	s	u
0–3	−0.060	−0.020	−0.006	−0.002	0	0	+0.004	+0.006	+0.014	+0.018
3–6	−0.070	−0.030	−0.010	−0.004	0	+0.001	+0.008	+0.012	+0.019	+0.023
6–10	−0.080	−0.040	−0.013	−0.005	0	+0.001	+0.010	+0.015	+0.023	+0.028
10–14	−0.095	−0.050	−0.016	−0.006	0	+0.001	+0.012	+0.018	+0.028	+0.033
14–18	−0.095	−0.050	−0.016	−0.006	0	+0.001	+0.012	+0.018	+0.028	+0.033
18–24	−0.110	−0.065	−0.020	−0.007	0	+0.002	+0.015	+0.022	+0.035	+0.041
24–30	−0.110	−0.065	−0.020	−0.007	0	+0.002	+0.015	+0.022	+0.035	+0.048
30–40	−0.120	−0.080	−0.025	−0.009	0	+0.002	+0.017	+0.026	+0.043	+0.060
40–50	−0.130	−0.080	−0.025	−0.009	0	+0.002	+0.017	+0.026	+0.043	+0.070
50–65	−0.140	−0.100	−0.030	−0.010	0	+0.002	+0.020	+0.032	+0.053	+0.087
65–80	−0.150	−0.100	−0.030	−0.010	0	+0.002	+0.020	+0.032	+0.059	+0.102
80–100	−0.170	−0.120	−0.036	−0.012	0	+0.003	+0.023	+0.037	+0.071	+0.124
100–120	−0.180	−0.120	−0.036	−0.012	0	+0.003	+0.023	+0.037	+0.079	+0.144
120–140	−0.200	−0.145	−0.043	−0.014	0	+0.003	+0.027	+0.043	+0.092	+0.170
140–160	−0.210	−0.145	−0.043	−0.014	0	+0.003	+0.027	+0.043	+0.100	+0.190
160–180	−0.230	−0.145	−0.043	−0.014	0	+0.003	+0.027	+0.043	+0.108	+0.210
180–200	−0.240	−0.170	−0.050	−0.015	0	+0.004	+0.031	+0.050	+0.122	+0.236
200–225	−0.260	−0.170	−0.050	−0.015	0	+0.004	+0.031	+0.050	+0.130	+0.258
225–250	−0.280	−0.170	−0.050	−0.015	0	+0.004	+0.031	+0.050	+0.140	+0.284
250–280	−0.300	−0.190	−0.056	−0.017	0	+0.004	+0.034	+0.056	+0.158	+0.315
280–315	−0.330	−0.190	−0.056	−0.017	0	+0.004	+0.034	+0.056	+0.170	+0.350
315–355	−0.360	−0.210	−0.062	−0.018	0	+0.004	+0.037	+0.062	+0.190	+0.390
355–400	−0.400	−0.210	−0.062	−0.018	0	+0.004	+0.037	+0.062	+0.208	+0.435

Surface finish:

By definition, surface finish is the allowable deviation from a perfectly flat surface that is made by some manufacturing process. All machining processes will produces some roughness on the surface. This roughness can be caused by a cutting tool, cutting rate and environmental conditions and the type of material you are working with. Surface texture consists of the repetitive and/or random deviations from the nominal surface of an object.

Surface finish (as shown in the figure below) is generally broken up into four components such as roughness, waviness, lay and flaws.



Roughness is generally the machined marks made on a surface by the cutting tool. It refers to the small, finely spaced deviations from the nominal surface that are determined by the material characteristics and the process that formed the surface.
Waviness. is defined as the deviations of much larger spacing; they occur because of work deflection, vibration, heat treatment, and similar factors. Roughness is superimposed on waviness.
Lay is the predominant direction or pattern of the surface texture. It is determined by the manufacturing method used to create the surface, usually from the action of a cutting tool. Figure below presents most of the possible lays a surface can take, together with the symbol used by a designer to specify them.

Lay symbol	Surface pattern	Description	Lay symbol	Surface pattern	Description
=		Lay is parallel to line representing surface to which symbol is applied.	C		Lay is circular relative to center of surface to which symbol is applied.
⊥		Lay is perpendicular to line representing surface to which symbol is applied.	R		Lay is approximately radial relative to the center of the surface to which symbol is applied.
X		Lay is angular in both directions to line representing surface to which symbol is applied.	P		Lay is particulate, nondirectional, or protuberant.

Flaws are irregularities that occur occasionally on the surface; these include cracks, scratches, inclusions, and similar defects in the surface. Although some of the flaws relate to surface texture, they also affect surface integrity.

Surface Finish Measurements and Charts:

(Source: Wikipedia: http://en.wikipedia.org/wiki/Surface_finish)
The parameters of texture are vertical amplitude variations, horizontal spacing variations, or some hybrid combination of these. All four surface finish components exist simultaneously. They simply overlap one another. We often look at each

(roughness, waviness, and lay) separately, so we make the assumption that roughness has a shorter wavelength than waviness, which in turn has a shorter wavelength than form.

Surface roughness is a measurable characteristic based on the roughness. Surface finish is a more subjective term denoting smoothness and general quality of a surface. In popular usage, surface finish is often used as a synonym for surface roughness. The most commonly used measure of surface texture is surface roughness. With respect to Figure below, surface roughness can be defined as the average of the vertical deviations from the nominal surface over a specified surface length. An arithmetic average (R_a) is generally used, based on the absolute values of the deviations, and this roughness value is referred to by the name average roughness. The R_a method is the most widely used averaging method for surface roughness today. An alternative is the root-mean-square (RMS) average, which is the square root of the mean of the squared deviations over the measuring length. RMS surface roughness values will almost always be greater than the R_a values because the larger deviations will figure more prominently in the calculation of the RMS value.

Surface roughness suffers the same kinds of deficiencies of any single measure used to assess a complex physical attribute. For example, it fails to account for the lay of the surface pattern; thus, surface roughness may vary significantly, depending on the direction in which it is measured. Another deficiency is that waviness can be included in the R_a computation. To deal with this problem, a parameter called the cutoff length is used as a filter that separates the waviness in a measured surface from the roughness deviations. In effect, the cutoff length is a sampling distance along the surface. A sampling distance shorter than the waviness width will eliminate the vertical deviations associated with waviness and only include those associated with roughness. The most common cutoff length used in practice is 0.8mm(0.030 in). The measuring length L_m is normally set at about five times the cutoff length. The limitations of surface roughness have motivated the development of additional measures that more completely describe the topography of a given surface. These measures include three-dimensional graphical renderings of the surface.

Depending on conventions in different countries, industries, applications, etc. the units used to express surface finish or roughness will vary. Likewise, various industry standards are used to specify the degree of roughness allowed or recommended in different applications. These standards include those published by **ANSI, ASME, SAE, ISO**, and other organizations.

Commonly used expressions of finish include:

Standard grit reference - refers to the grit of a surface finishing medium or method, which does not provide a consistent measure of roughness, since results depend on a part's material, finishing method, lubricant used (if any), and applied work pressure.

N- New ISO (Grade) Scale numbers. These are used on manufacturing drawings that specify surface finish in terms of an ISO standard. Each roughness grade number can be correlated to a specific R_a number that is expressed in microns.

R_a- Roughness average, most commonly expressed in micrometers (microns). This is the most universally recognized and used international standard of roughness measurements. It is the arithmetic mean of the absolute departures of a roughness profile from the mean line of the measurement. R_a may also be expressed in microinches.

R_p- Maximum profile peak height.

R_v- The deepest valley below the mean line.

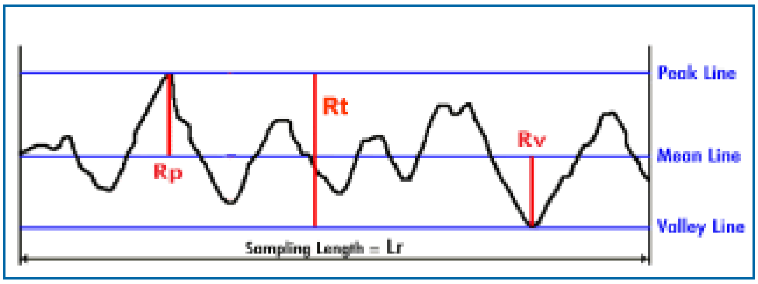
R_t- The total height of a roughness profile, typically expressed in microns, is the maximum peak-to-valley height along the assessment length.
R_z - The average **R_t** over a given length.

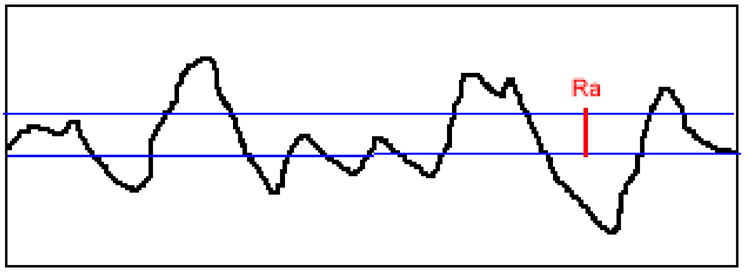
$$R_z=(R_{p1}+R_{p2}+R_{p3})+(R_{v1}+R_{v2}+R_{v3}) / 3$$

CLA- Center Line Average in micro-inches. This is a conversion using Ra(µm) x 39.37.
RMS- Root Mean Square in micro-meters or micro-inches; i.e., the average of peaks and valleys of a material's surface profile as calculated from a number (n) of measurements (x) along the sampling length:

$$x_{\mathrm{rms}} = \sqrt{\frac{1}{n}\left({x_1}^2 + {x_2}^2 + \cdots + {x_n}^2\right)}$$

RSm- The mean spacing between profile peaks on the mean line, measured along the sampling length.





Most expressions of roughness can be converted from one form to another. For example, CLA (microinches) = R_a(µm) x 39.37(inches/meter)
Other conversions use factors that have been establish as generally acceptable over time. In the case of RMS, a range of factor values from 1.1 to 1.7 can be acceptable. A factor of 1.1 is probably use most often, i.e., RMS(µin.) = CLA(µin.) x 1.1. Table 1 lists conversions for some commonly used roughness expressions and values.

Table 1. Conversion chart for equivalent expressions of roughness.

Grit No.	ISO No.	R_a (µm)	R_a (µin.)	CLA (µin.)	RMS (µin.)[1]	R_t (µm)[2]
-----------	**N12**	50	2000	2000	2200	200
-----------	**N11**	25	1000	1000	1100	100
-----------	**N10**	12.5	500	500	550	50
60	**N9**	6.30	250	250	275	25
-----------	**N8**	3.20	125	125	137.5	13
80	-----------	1.80	71	71	78	9.0
-----------	**N7**	1.60	63	63	64.3	8.0
120	-----------	1.32	52	52	58	6.6
150	-----------	1.06	42	42	46	5.3

-----------	**N6**	0.80	32	32	32.5	4.0
180	**-----------**	0.76	30	30	33	3.8
220	**-----------**	0.48	19	19	21	2.4
-----------	**N5**	0.40	16	15	17.6	2.0
240	**-----------**	0.38	15	12	17	1.9
320	**-----------**	0.30	12	9	14	1.5
400	**-----------**	0.23	9	8	10	1.3
-----------	**N4**	0.20	8	4	8.8	1.2
500	**N3**	0.10	4	2	4.4	0.8
-----------	**N2**	0.05	2	1	2.2	0.5
-----------	**N1**	0.025	1	1	1.1	0.3

Notes: 1. A factor of 1.1 X CLA is used throughout this table to calculate RMS(µin.)
2. Typically, for values of R_a from 50µm to 3.2µm, the conversion factor for R_t (µm) is 4. As surface roughness decreases from 3.2µm, the conversion factor increases, reaching 12 at 0.025µm. This is reflected in the table above.

Surface Finish Affects Performance

The surface finish of process vessels, piping and related components can be a critical factor in their performance, maintenance costs, and service life. Until recently, specifying and measuring surface finish involved varying degrees of speculation. Today, it is more likely that this characteristic will be influenced by industry standards, which manufacturers and processors must satisfy.

Increasingly stringent specifications are creating greater demand for improved surface finish on most metal components that are part of process equipment. In particular higher purity requirements for pharmaceutical and biotechnology products are dictating the characteristics of surfaces in contact with process fluids. Increasingly, process equipment components must meet requirements in the ASME Bio-processing equipment standard, ASME-BPE-2009.

This standard provides specifications for the design, manufacture and acceptance of vessels, piping and related components for application in equipment used by the biotechnology, pharmaceutical, and personal care product industries. It includes aspects related to sterility and cleanability, materials, dimensions and tolerances, surface finish, material joining, and seals. Meeting the surface finish requirements of this standard is rapidly becoming a universal necessity in the manufacture of other fluid process equipment. As a result, suppliers of equipment and components are often required to quantify the surface roughness of their finished products.

Some additional standards and specifications that directly or indirectly affect surface finish requirements include:

ASME B46.1-2002 - Surface Roughness, Waviness, and Lay
ISO 4287 and 4288 - Geometrical Product Specifications **(GPS)**
DIN ISO 1302, **DIN** 4768 - Comparison of Roughness Values
ASME Y14.36M - Surface Texture Symbols
ASME B16.5 - Pipe Flange Face Roughness
DIN 7079 Standard for Fused-Glass Sight Glasses in Metal Frames

Such standards have come into play because process engineers realize that the surface finish of vessels, piping and related components can have profound effects on how well a fluid system performs. Typically, surface roughness is a critical parameter in the assessment of surface finish on fluid system components. This parameter can affect fluid flow resistance (friction), adsorption/desorption, the build-up of chemicals from a process fluid, corrosion formation, pressure drop, etc. Ultimately, surface finish can affect service life and maintenance costs.

Surface Finish on Engineering Drawings:

Symbols for Surface Texture Designers specify surface texture on an engineering drawing by means of symbols looks like a square root sign as shown. The symbol designating surface texture parameters is a check mark, with entries as indicated for average roughness, waviness, cutoff, lay, and maximum roughness spacing.

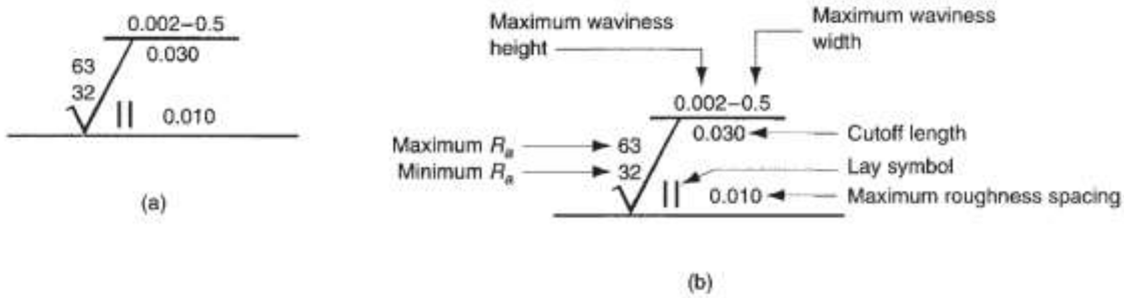


FIGURE 5.16 Surface texture symbols in engineering drawings: (a) the symbol, and (b) symbol with identification labels. Values of R_a are given in microinches; units for other measures are given in inches. Designers do not always specify all of the parameters on engineering drawings.

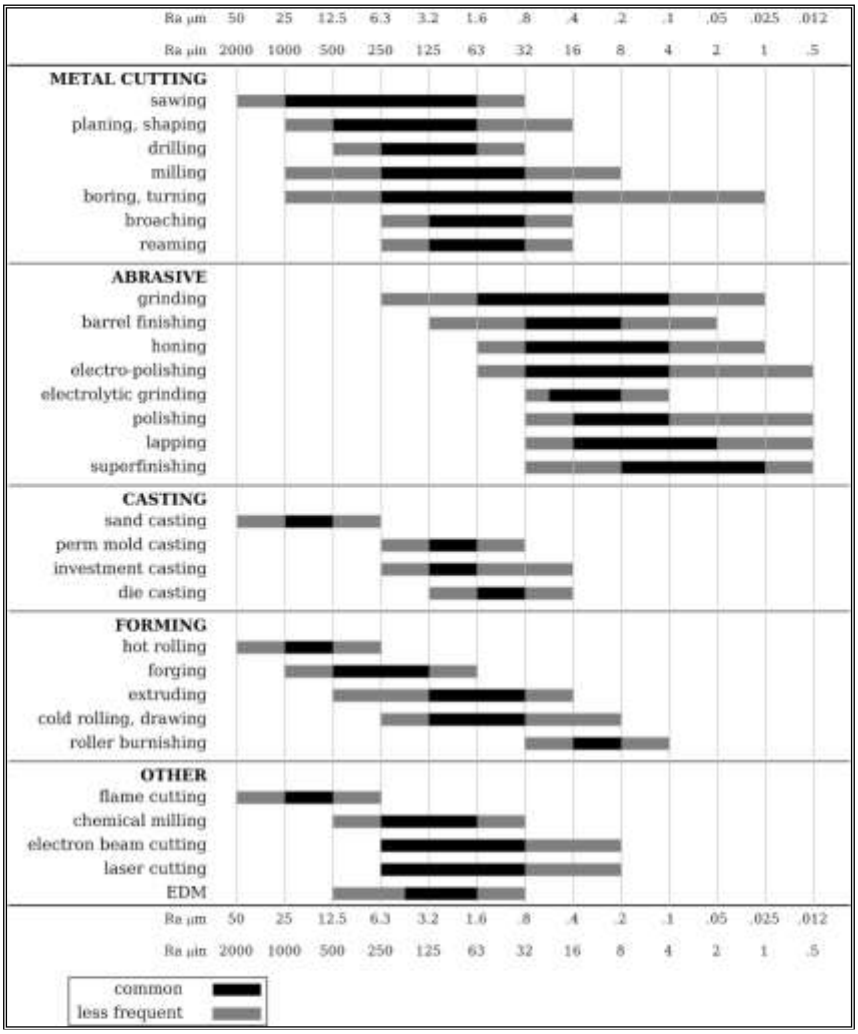
Relation between manufacturing processes, tolerances and surface finish

Many factors contribute to the surface finish in manufacturing. In forming processes, such as molding or metal forming, surface finish of the die determines the surface finish of the workpiece. In machining the interaction of the cutting edges and the microstructure of the material being cut both contribute to the final surface finish. In general, the cost of manufacturing a surface increases as the surface finish improves.

Just as different manufacturing processes produce parts at various tolerances, they are also capable of different roughnesses. Generally these two characteristics are linked: manufacturing processes that are dimensionally precise create surfaces with low roughness. In other words, if a process can manufacture parts to a narrow dimensional tolerance, the parts will not be very rough.
Due to the abstractness of surface finish parameters, engineers usually use a tool that has a variety of surface roughnesses created using different manufacturing methods.
Primary manufacturing processes establish the initial surface characteristics of components and their roughness values. In the case of metallic components, additional finishing processes may be used to reduce the degree of roughness to fit a specific application. Table 2 lists typical R_a values for various metal finishing methods. In the case of fluid system components, the motivation to reduce surface roughness could be to reduce flow resistance and pressure drop, improve sealing, reduce build-up of process chemicals on the metal surface, improve corrosion resistance to increase life, etc. In sight glasses, for example, the surface roughness of

both the glass and the metal mounting ring are critical for achieving a good seal in the installation.

Table 2. Typical range of R_a surface roughness values in various metal forming operations



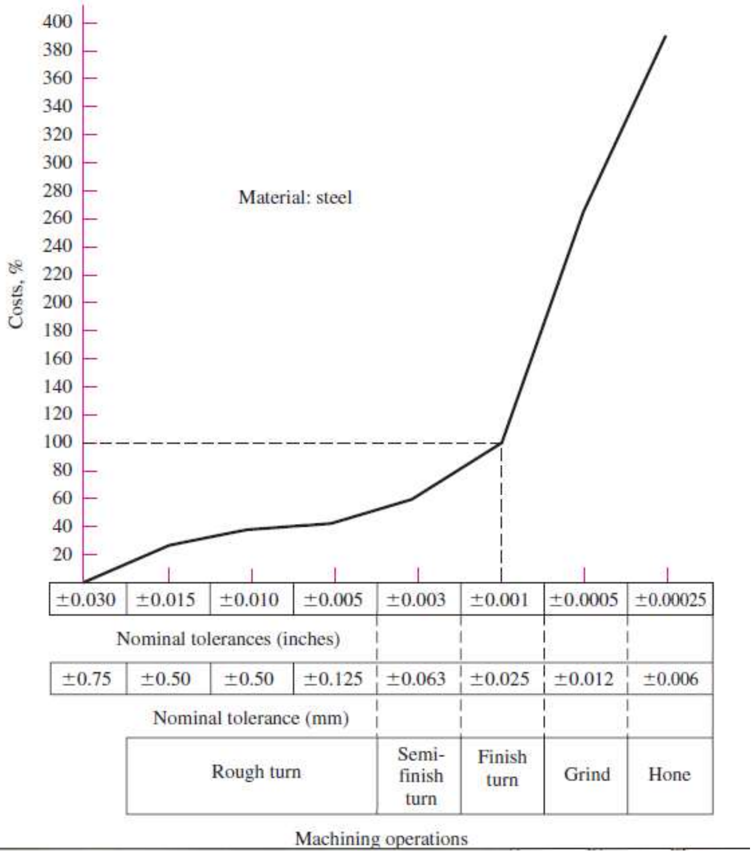
Various types of polishing operations are commonly used to reduce the surface roughness of metals used in fluid vessels, piping and related components. These fall into two categories: mechanical polishing and electropolishing. As the name implies, mechanical polishing involves the application of physical force on abrasive media to remove surface irregularities. While it's theoretically possible to achieve low roughness values with certain mechanical polishing techniques, the time and cost involved usually makes this impractical. Generally, mechanical polishing is used when moderate roughness values are acceptable, which means numerous surface scratches and other irregularities remain. These can cause many of the problems mentioned earlier on this page.

Electropolishing is an electrolytic process (the reverse of plating) combining electric current and chemicals to remove metal. The peaks of burr, folds, inclusion and other anomalies of a metal surface are dissolved more quickly than valleys as a result of the greater concentration of current over the protuberances. This electrochemical action produces a smoothing and rounding of the surface profile, resulting in irregularities as small as 0.01 micrometer (0.04 micro-inch). It prevents or reduces most of the problems associated with rougher metal surfaces. The inherent benefits of electropolishing subsequent to mechanical polishing include:

- Removal of surface occlusions
- Removal of inclusions and entrapped contaminants such as lubricants and grit particles
- Cleaner surface of the a wet contact areas
- Reduced surface area/chemical reactivity for less absorption and adsorption
- Less contamination and build-up of process chemicals on a surface
- Superior surfaces for cleaning and sterilization
- Elimination of localized corrosive cells (galvanic differences) remaining after mechanical polishing
- Resultant passivated surfaces enhance corrosion resistance
- High luster reflective appearance
- Reduced surface friction

Figure 1–2

Cost versus tolerance/machining process.
(From David G. Ullman, The Mechanical Design Process, 3rd ed., McGraw-Hill, New York, 2003.)

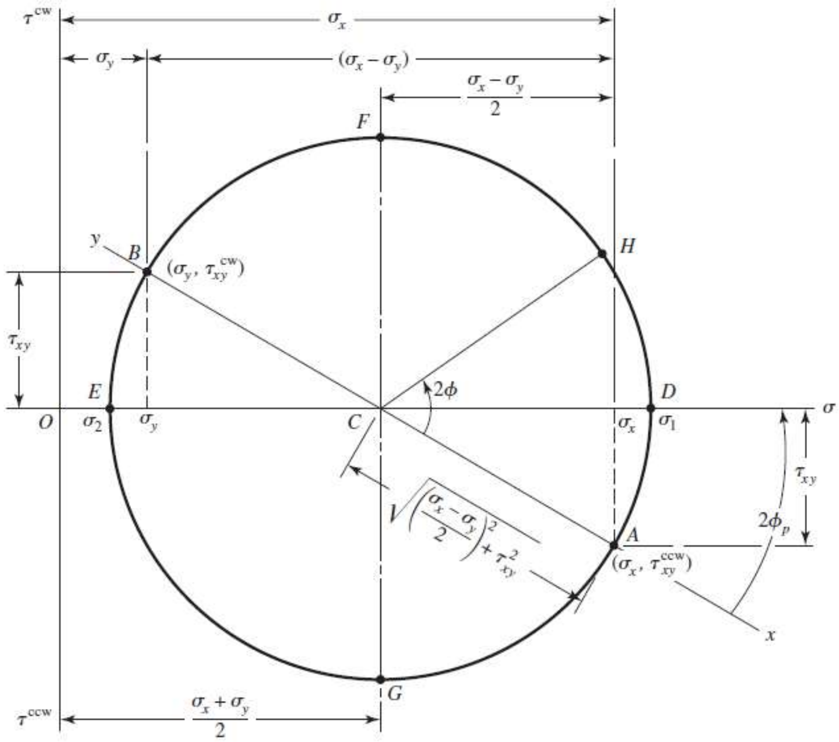


Part II

Stress Analysis

This part includes:

- ✓ **Simple Stresses**
- ✓ **Types of Loading**
- ✓ **Combined stresses**
- ✓ **Failure theories**



Simple Stresses:

Most mechanical loading, whatever its complexity, convert at the end to three simple types of stress. These three types are normal, shear and bearing stresses and they will be explained in the following paragraphs and example.

Tensile and compressive stresses, called normal stresses are tend to pull on or crush the element. For a load-carrying member in which the external load is uniformly distributed across the cross-section area of the member, the magnitude of the stress can be found by:

$$\boldsymbol{\sigma_{tp} = force/area = P/A}$$

where σ_{tp} is the permissible (allowable) tensile stress that can be found from tables or Tensile Test in the following sense.

$$\boldsymbol{\sigma_{tp} = \frac{\sigma_y}{n} = \frac{\sigma_{ult}}{N}}$$

where σ_y is the yield stress, σ_{ult} is the ultimate stress, n is the safety factor based on yield stress and N is the safety factor based on ultimate stress. N and n can be found as explain in the next section. Note that for most ductile materials the compressive strengths are about the same as the tensile strengths.

Selection of The Safety Factor

Selection of a design safety factor must be undertaken with care since there are unacceptable consequences associated with selected values that are either too low or too high. Engineers must accommodate uncertainty. Uncertainty always accompanies change. Material properties, load variability, fabrication fidelity, and validity of mathematical models are among concerns to designers. To implement the selection of a design safety factor, consider separately each of the following eight factors:

1. The accuracy with loads, force, deflections or other failure-inducing agents can be determined.
2. The accuracy with which the stresses or other loading severity parameters can be determined from the force or other failure-inducing agents.
3. The accuracy with which the failure strengths or other measures of failure can be determined for the selected material in the appropriate failure mode.
4. The need to conserve material, weight, space, or money.
5. The seriousness of consequences of failure of human life and/or property damage.
6. The quality of workmanship in manufacture.
7. The conditions of operation.
8. The quality of inspection and maintenance available or possible during operation.

For this course, we will simplify the selection of the safety factor and we will use the following table that based and considered some factors which explained in the above method.

Type of load	**Ductile**		**Brittle**
	N	**N**	**N**
Static load	**3-4**	**1.5-2**	**5-6**
Repeated, Reversed (Mild shock)	**8**	**4**	**10-12**
Shock (Sudden)	**10-15**	**5-7**	**15-20**

Direct shear stress occurs when applied force tends to cut through the member as a scissors or shears do when a punch and a die are used to punch a slug of material from a sheet. Another important example of direct shear in machine is the tendency for a key to be sheared off at the section between shaft and the hub of machine element when transmitting torque. The method of computing direct shear is based that the applied force is assumed to be uniformly distributed across section of the part that is resisting the force.

$$\boldsymbol{\tau_p = Shearing\ force/area\, in\ shear = P/A_s}$$

where τ_p is the permissible (allowable) shear stress that can be found from shear test. Unfortunately, these values are seldom reported and if this test is not available we can estimate it as (0.5-0.66) of σ_{tp} for simplicity.

Bearing stresses, sometimes called contact stresses, are occurs when one surface crush the surface of other element. If we assume that the external load is uniformly distributed across the contact area of the two surfaces, the magnitude of the bearing stress can be found by:

$$\boldsymbol{\sigma_{bp} = force/projected\ area\ in\ bearing = P/A_b}$$

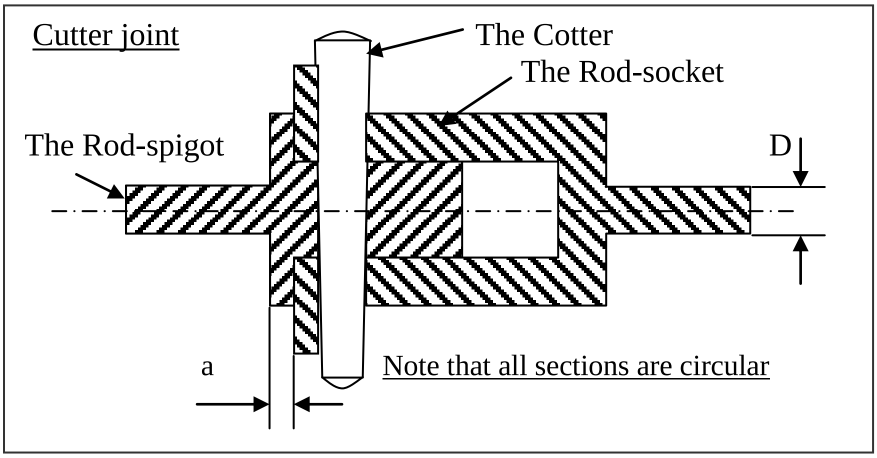
where σ_{bp} is the permissible (allowable) bearing stress that can be found from test and if this test is not available we can estimate it as 1.33 of σ_{tp} for simplicity.

The simple relation between different stresses is

$$\boldsymbol{\sigma_{tp} = \frac{3\,\sigma_{bp}}{4} = \frac{3\tau_p}{2}}$$

Case study (1): Design of the cutter joint:

The cutter joint is linking between two parts. The loading conditions are tension and compression with maximum force of P. This force is transferring from one end to the other. During this transformation, the stresses in each section will change from type to type. Some sections will carry normal stress and other will carry shear stress or bearing.



Types of loading

Axial load(tensile stress): This is the simplest loading conditions, and the stress is found by:

$$\sigma = \text{Force} / \text{Area}$$

Bending moment: In this loading condition the maximum stress is the tensile stress at the most faraway point from the neutral axis and equal:

$$\sigma = MY / I$$

Torsion stress: The shear stress can be found from the well-known torsion formula:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{l}$$

Combined stresses

The real life stresses are normally a combined stresses consists of one or more of the above stresses. The following cases are the main types.

One dimensional stresses:

If all the stresses are working on one dimension (shear only or normal stress only) the resultant stress will be calculated simply as the summation of all of them.

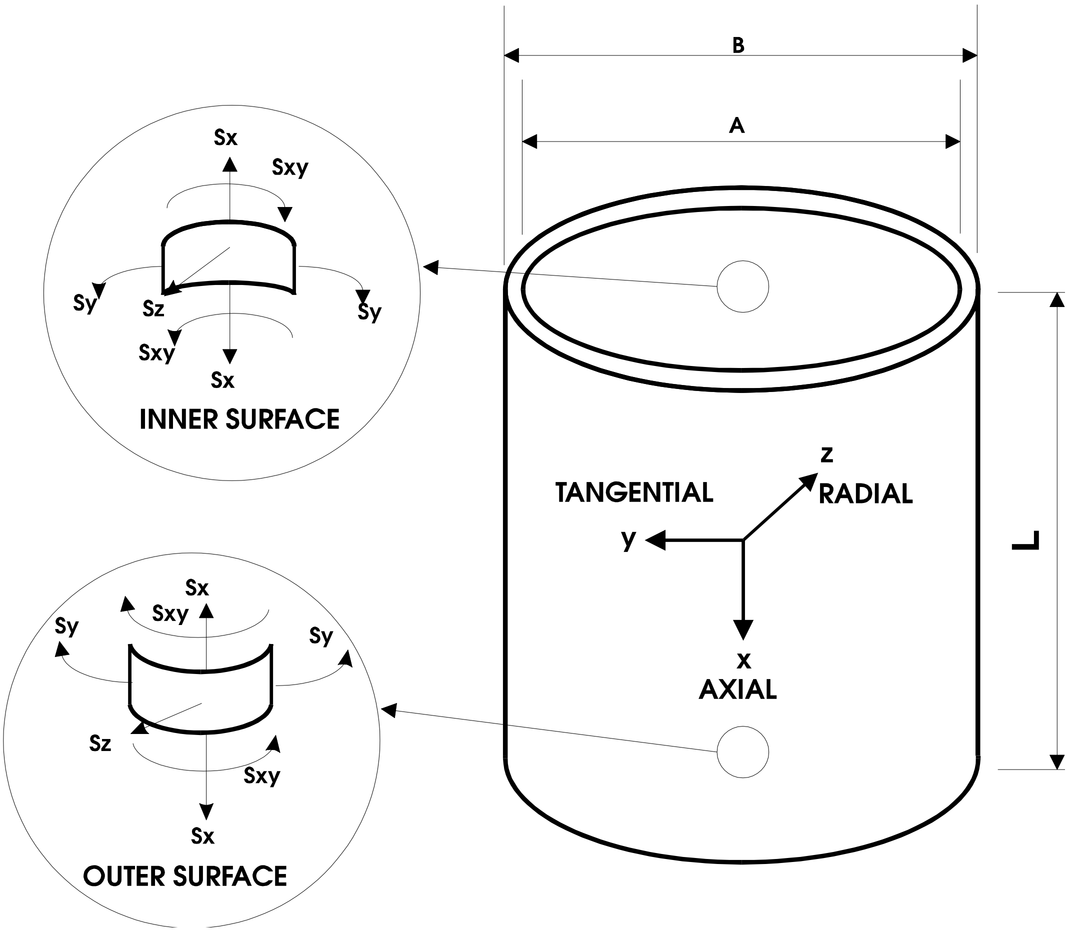
Two dimensional stresses:

In two dimensional stresses Mohr's circle is used to determine the maximum normal and shear stresses through the section. If two perpendicular normal stresses are applied in addition to shear stress between them, the principle and shear stresses are calculated as follows:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

Three dimensional stresses:



BASIC DIMENSION OF THE TUBE AND XYZ CO-ORDINATE SYSTEM SHOWING POSITIVE DIRECTION OF THE STRESSES.

In this loading conditions the following procedure can be used:

Step (1): The principle normal stresses are calculated by a procedure explained by many text books This procedure is based on finding a series of (11) constants where the last (3) are the principle normal stresses at the point analysed. These constants can be summarised as the following:

$$A1 = Sx + Sy + Sz$$

$$B1 = SxSy + SySz + SzSx - Sxy^2 - Syz^2 - Szx^2$$

$$C1 = SxSySz + 2SxySyzSzx - SxSyz^2 - SySzx^2 - SzSxy^2$$

$$D1 = A^2/3 - B$$

$$E1 = A\,B/3 - C - 2A^3/27$$

$$F1 = \sqrt{D^3/27}$$

$$G1 = \arccos(-E/2F)$$

$$H1 = \sqrt{D/3}$$

$$I1 = 2H\cos(G/3) + A/3$$

$$J1 = 2H\,[\cos(G/3 + 120^\circ)] + A/3$$

$$K1 = 2H\,[\cos(G/3 + 240^\circ)] + A/3$$

As a check, if the algebraic sum I1+J1+K1 equals A1, within rounding errors, then the calculations up to this point should be correct.
Step (2): I1, J1, and K1 are the maximum principal normal stresses.
Step (3): Calculate the true maximum shear stress by:

WSD = 0.5(PR1-PR2)

The largest value from I1, J1 and K1 is PR1 and alternatively the smallest is PR2.
Step (4): The maximum normal principal stresses and the maximum true shear stress can be used now with various theories of failure.

Failure theories:

The tables of the mechanical properties of material usually contain design information about a simple load case such as tensile or compression tests' results. In order to determine suitable allowable stresses for the complicated condition, like ours, various strength theories have been developed. Some of these theories will be summarised here.
The maximum stress theory (Rankine's theory): This theory states that failure will happen at the maximum or minimum of the principal stresses.
The maximum strain theory: This theory states that the failure happens at the place where the strain becomes equal to maximum strain in the simple load case.
The maximum shear theory: This theory assumes that failure begins when the shear stress in the material becomes equal to the maximum shear stress with a simple tensile test. Timoshenko (1956) showed that the agreement between this theory and the experiment is better especially with the ductile materials. This theory is simple to apply because the allowable shear stress is normally one half of the tensile stress and the actual maximum shear can be calculated by

$$\text{Max. Shear} = \frac{\text{Max. principal stress - Min. principal stress}}{2}$$

So,
Max. allowable stress = Max. principal stress -Min. principal stress
The maximum energy theory: In this theory the quantity of strain energy per unit volume of the material is used as the basis for determining failure. This can be explained mathematically as:

$$\sigma_{all.(\max)} = \sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\nu(\sigma_1\sigma_2 + \sigma_3\sigma_3 + \sigma_1\sigma_3}$$

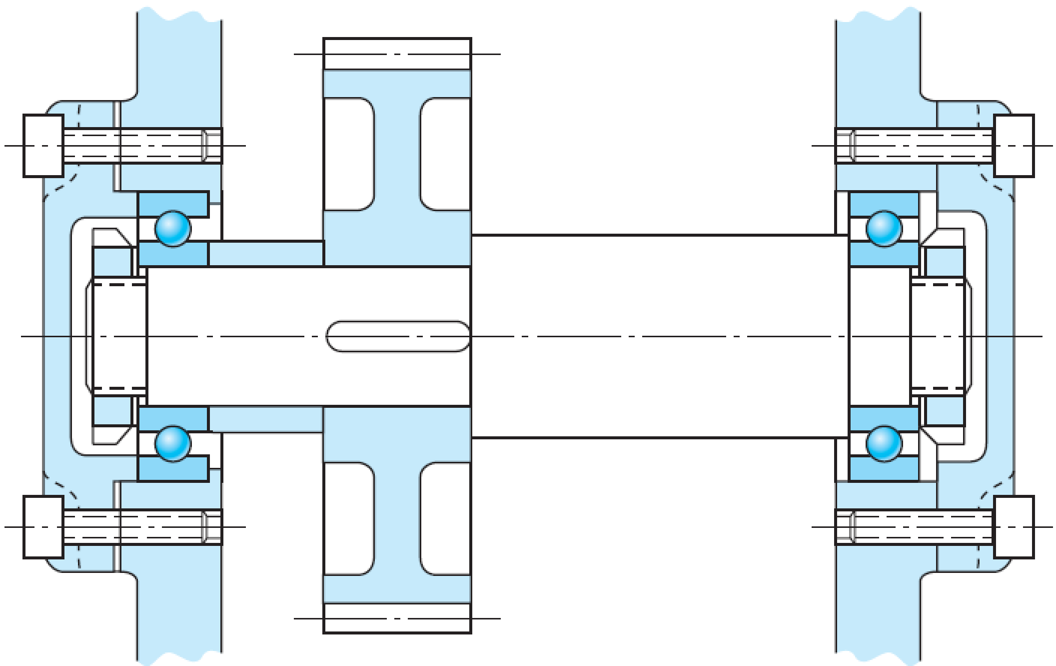
Where σ *all.* (max.) is the maximum allowable stress and σ_1, σ_2 and σ_3 are the normal principal stresses.
By reviewing many mechanical design references it was found that the maximum shear theory is recommended for use with ductile materials and the maximum energy theory with the brittle materials. This recommendation is used in this work.

Part III

Design of Mechanical Elements

This part includes:

- ✓ **Shafts, axles and their components**
- ✓ **Bearings**
- ✓ **Couplings**
- ✓ **Screws**
- ✓ **Welding and Riveted joints**
- ✓ **Mechanical Springs**
- ✓ **Clutches**
- ✓ **Brakes**
- ✓ **Belt**
- ✓ **Chain**
- ✓ **Gears**



Shaft and axle

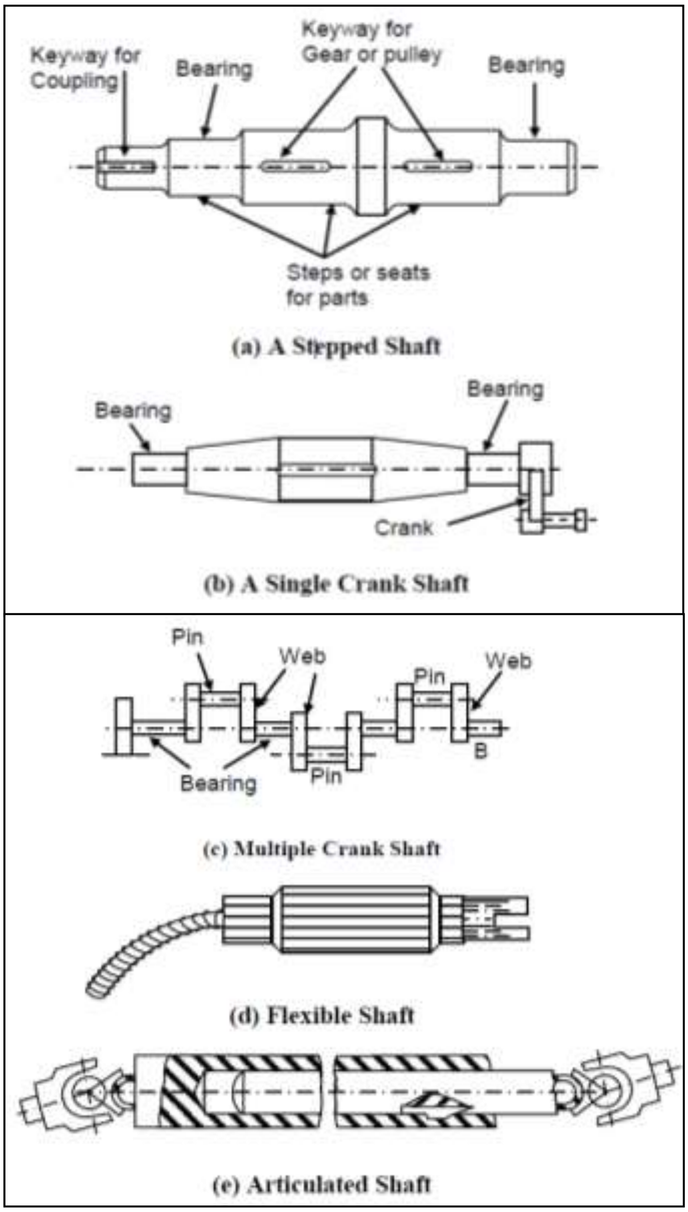
Shafts are important elements of the machines. They are the elements that support rotating parts like gears and pulleys and in turn are themselves supported by bearings resting in the rigid machine housings. The **shafts** perform the function of transmitting power from one rotating member to another supported by it or connected to it. Thus, they are subjected to torque due to power transmission and bending moment due to reactions on the members that are supported by them. Shafts are to be distinguished from **axles** which also support rotating members but do not transmit power. Axles are thus subjected to only bending loads and not to the torque.

Most the times, shafts have circular cross-section and could be either solid or hollow. The shafts are classified as straight, cranked, cam, flexible or articulated. Straight shafts are commonest to be used for power transmission. Such shafts are commonly designed as stepped cylindrical bars, that is, they have various diameters along their length, although constant diameter shafts would be easy to produce. The stepped shafts correspond to the magnitude of stress which varies along the length. Moreover, the uniform diameter shafts are not compatible with assembly, disassembly and maintenance. Such shafts would complicate the fastening of the parts fitted to them, particularly the bearings, which have to be restricted against sliding in axial direction. While determining the form of a stepped shaft it is borne in mind that the diameter of each cross-section should be such that each part fitted on to the shaft has convenient access to its seat.

The parts carried by **axle or shaft** are fastened to them by means of keys or splines and for this purpose the shaft and axle are provided with key ways or splines. The bearings that support the shafts or axle may be of sliding contact or rolling contact type. In the former case the journal of the shaft rotates freely on thin lubricant layer between itself and bearing, while in the latter case the inner race of the bearing is force fitted on the journal of the shaft and rotates with the shaft while outer race is supported in the housing and remains stationary.

A shaft is joined with another in different ways and configurations. The coaxial shafts are connected through couplings which may be rigid or flexible.

Types of shaft: the types of shaft are shown in the figure that shows (in part a) a stepped shaft with three seats for supported parts which can be pulleys, gears or coupling. Two seats for bearings are also indicated. These bearings will be rolling contact type. Figure (b) shows a single crank shaft. The crank may be connected to another element like



(a) A Stepped Shaft

(b) A Single Crank Shaft

(c) Multiple Crank Shaft

(d) Flexible Shaft

(e) Articulated Shaft

connecting rod which may have a combined rotary and reciprocating motion. The connection is through a bearing often called crank pin. The straight part of the shaft may support a pulley or a gear. The connection will be through a key. Multiple crank shaft is shown in Figure (c). Each crank pin would carry a connecting rod and each crank pin will be between the supporting bearings. The other shaft types are explanatory.
The adjacent sections of shafts with different diameters are joined by smooth transition fillet with as large radius as permitted by supported part or bearing that supports the shaft. The larger radius of fillets will reduce stress concentration factor.
Materials for shafts: From the above discussion the materials for the shaft would be required to possess
(a) high strength,
(b) low notch sensitivity,
(c) ability to be heat treated and case hardened to increase wear resistance of journals, and
(d) good machinability.

Shafts could be made in mild steel, carbon steels or alloy steels such as nickel, nickel-chromium or chrome-vanadium steels.
The following Table describes shafting available sizes commercially.

Standard Sizes of Commercial Shafting (Diameter)
Up to 25 mm in increment of 0.5 mm
25 to 50 mm in increment of 1.0 mm
50 to 100 mm in increment of 2.0 mm
100 to 200 mm in increment of 5.0 mm

<u>Shaft design</u>:
Shaft is an important machine element and transmits power. Shafts are many types (see the figure above) and are made, mainly, cylindrical. They are subjected to torque and bending moment, hence, at any point in the section of shaft there exists direct bending stress due to bending moment and shearing stress due to torque. They are designed against maximum principal stress or maximum shearing stress. The load (comprising bending moment and torque) is converted into equivalent bending moment or equivalent torque. The diameters are calculated by modifying the expressions for equivalent bending moment and equivalent torque by considering condition and manner of loading. The keyways become essential feature of shafts because some part like gear or pulley has to be attached on it to transmit power. The keys are standardized and can be selected from relevant table. Alloy steel shafts are not uncommon if corrosive atmosphere exists. Cast iron shaft, though used rarely, will tend to become heavier.

In general, shaft design is effected by the kind of the loading that depends on the connecting parts like Pulley, spur gear, helical gear, or bevel gear. It is not necessary to evaluate the stresses in a shaft at every point; a few potentially critical locations will suffice. Critical locations will usually be on the outer surface, at axial locations where the bending moment is large, where the torque is present, and where stress concentrations exist. By direct comparison of various points along the shaft, a few critical locations can be identified upon which to base the design. An assessment of typical stress situations will help.
Shafts can be designed according to:

1. **Strength:** In this method we find the dimensions of the shaft that allow it to work under stress level less than the maximum allowable stresses

2. **Rigidity :** In this method we find the dimensions of the shaft that allow it to deflect in the range of allowable deflections)

Design according to the strength

The following cases will be discussed:

1. Shafts subjected to twisting moment only.
2. Shafts subjected to bending moment only.
3. Shafts subjected to twisting moment and bending moment.
4. Shafts subjected to twisting moment, axial force, and bending moment.

1. Shafts subjected to twisting moment only:

Torsion formula will be used directly to find the dimensions according to the working shear stress.

$$\tau = \frac{T\,r}{J} = \frac{16\,T}{\pi\,d^3}$$

where τ is the allowable shear stress, T is the torsional torque, J is the polar moment of inertia, r is the shaft radius and d is the shaft diameter.

2. Shafts subjected to bending moment only:

Bending moment formula will be used to find the shaft dimension that can carry the maximum normal stress.

$$\sigma = \frac{M\,Y}{I} = \frac{32\,M}{\pi\,d^3}$$

where σ is the allowable normal stress, M is the bending moment, I is the moment of inertia, Y is the shaft radius and d is the shaft diameter.

3. Shafts subjected to twisting moment and bending moment:

$$\tau = \frac{T\,r}{J} = \frac{16\,T}{\pi\,d^3}$$

$$\sigma = \frac{M\,Y}{I} = \frac{32\,M}{\pi\,d^3}$$

<u>A. Using the maximum shear theory:</u>

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$\tau_{max} = \frac{16}{\pi d^3}T_e \quad \text{where } T_e = \sqrt{M^2 + T^2}$$

B. Using the maximum normal stress theory

$$\sigma_{max} = \frac{1}{2}\sigma + \sqrt{\left(\frac{1}{2}\sigma\right)^2 + (\tau)^2}$$

$$\sigma_{max} = \frac{1}{2}\left(\frac{32\,M}{\pi d^3}\right) + \sqrt{\left[\frac{1}{2}\left(\frac{32M}{\pi d^3}\right)\right]^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$\sigma_{max} = \frac{32}{\pi d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

$$\sigma_{max} = \frac{32}{\pi d^3}M_e \text{ where } M_e = \left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

Note:
All of the above equations used for sold circular cross section shafts. For hollow shafts with internal diameter of **D** and external diameter **d** of we use:

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16}\tau\; d^3(1 - K^4)$$

$$M_e = \left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right] = \frac{\pi}{32}\sigma\; d^3(1 - K^4)$$

$$\text{Where } K = \left(\frac{D}{d}\right)$$

Shafts subjected to fluctuating load:

In order to design such shafts, the combined shock and fatigue factor must be taken in account. Thus the shafts subjected to bending and torsion:

$$T_e = \left\{\sqrt{(k_m M)^2 + (k_t T)^2}\right\}$$

$$M_e = \left\{\frac{1}{2}\left[(k_m M) + \sqrt{(k_m M)^2 + (k_t T)^2}\right]\right\}$$

Where k_m combined shock and fatigue factor for bending, and k_t combined shock and fatigue factor for torsion. These factors can be choose according the following table:

No.	Nature of the load	k_m	k_t
1	**Stationary shafts:** a) gradually applied load b) suddenly applied load	 **1.0** **1.5 to2**	 **1.0** **1.5to 2**
2	**Rotating shafts:** a) gradually applied load Steady load b) suddenly applied load Minor shock Heavy shock	 **1.5** **1.5** **1.5 to 2** **2.0 to 3**	 **1.0** **1.0** **1.5 to 2** **1.5 to 3**

4. Shafts subjected to twisting moment, axial force, and bending moment:
Stress due to axial load (**F**) for round solid shaft is:

$$\sigma = \frac{4F}{\pi d^2}$$

Stress due both axial load and bending moment is:

$$\sigma = \frac{4F}{\pi d^2} + \frac{32M}{\pi d^3} = \frac{32}{\pi d^3}\left(M + \frac{Fd}{8}\right) = \frac{32M_r}{\pi d^3}$$

$$M_r = \left(M + \frac{Fd}{8}\right)$$

Buckling effect:
In the case of long shafts subjected to compressive load **F** a factor α must be introduced to take the column effect into account:

$$\sigma = \alpha \frac{4F}{\pi d^2}$$

Where α is column factor

$$\alpha = \frac{1}{1 - 0.00044\frac{L}{k}} \quad \text{where } \frac{L}{k} < 115$$

$$\alpha = \frac{\sigma_y}{C\pi^2 E}\frac{L}{k} \quad \text{where } \frac{L}{k} \geq 115$$

Where: **L:** Length of shaft between bearing.
k: Least radiuses of gyration
σ_y**:** Compressive yields stress in shaft
E: Modules of elasticity
C: Coefficient in Euler's formula depending on the end conditions as in the following table:

Conditions	C
Both ends hinged	**1**
Both ends fixed	**4**
One end fixed and the other hinged	**2**
One end fixed and the other free	**0.25**

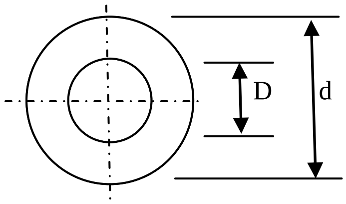
The general set of equations: Finally, we reach the most general equations to find the equivalent torsional torque and bending moment of the hollow circular shaft as:

$$T_e = \sqrt{\left(k_m M + \frac{\alpha F d (1 - K^2)}{8}\right)^2 + (k_t T)^2} = \frac{\pi}{16}\tau\ d^3(1 - K^4)$$

$$\mathbf{M_e} = \frac{1}{2}\left[\left(\mathbf{k_m M} + \frac{\alpha\,\mathbf{F\,d}\,(1-\mathbf{K}^2)}{8}\right) + \sqrt{\left(\mathbf{k_m M} + \frac{\alpha\,\mathbf{F\,d}\,(1-\mathbf{K}^2)}{8}\right)^2 + (\mathbf{k_t T})^2}\,\right]$$

$$= \frac{\pi}{32}\sigma\ \mathbf{d}^3(1-\mathbf{K}^4)$$

Where $\mathbf{K} = \left(\frac{D}{d}\right)$



Stress concentration

Shaft must, in most cases, have shoulder, key way, holes, oil groves and notches of various kinds. Any discontinuity alters the stress distribution in the neighborhood area. Such discontinuities are called stress raisers and the regions in which they occur called stress concentration.

Stress-concentration factors for a variety of geometries may be found in tables A-15 and A-16.

The value of allowable stress may be further reduced by 25% if keyway is present

As designers, we need to understand that the potential for stress concentrations to produce fatigue cracking can be reduced in two ways.

- Reduce the stress-concentration effect by making the change of shape more gradual.
- Relocate the stress concentration or change of shape to an area subjected to lower stresses.

Design according to the rigidity

Shafts are often designed for strength as illustrated in theory so far. But all shafts have to be stiff and rigid so that their deflection and twist are within permissible limits. If the shaft exceeds in deflection and twist limits the diameter has to be increased. We must remember that the deflection and twists are inversely proportional to cube of the diameter hence, lesser diameter will result in greater deflection and twist. The problem becomes important when high strength steel is used for shaft. Such shaft will result in smaller diameter and hence, larger deflection. Moreover, using high strength steel requires greater care for its greater notch sensitivity. The permissible values of displacement (in bending and torsion) are decided with respect to the requirements of machine in which shaft is placed, hence, such values vary from machine to machine. For example, permissible deflection of shaft in machine tool may depend upon module of the gear fitted on the shaft while the limit in shaft of the rotor of an electric motor will be in function of air gap. In general, however, the maximum deflection in shaft must not exceed 0.2% of the span between the bearings in case of machines with gears mounted on shafts. The slope due to bending at the bearings must also be limited. Following are the limits for precision machines :

Slope ≤ 0.001 rad if bearing sliding contact type.

Slope ≤ 0.008 rad if bearing rolling contact type.

Slope ≤ 0.050 rad if bearing self aligning type.

The angular twist may become basic design consideration for shaft such as in drilling machine where the twist should not be greater than 0.035 radius over a length of 25 × diameter. The transmission shaft in a gantry crane is not allowed to twist more than 0.012 rad per meter length.

In general, the deflection of shaft is reduced by

(a) making mounted parts lighter,

(b) keeping mounted parts balanced, and

(c) mounting parts close to bearing.

For simplification purpose, two types of rigidity can be defined:

1. Torsional rigidity (**Φ** not more than)

Two permissible amount of twist:

A. $\leq$ 0.25° per one meter

B. $\leq$ 1° per 20 diameters

2. Lateral rigidity (deflection not more than)

It must be consistent with the permissible lateral deflection for proper bearing clearance and for correct gear teeth alignment. Deflection can be found by the standard ways.

Summary:

Shaft Design Procedure

• Develop a static free-body diagram.
• Draw a bending moment diagram in two planes.
• Develop a torque diagram.
• Establish the location of the critical cross section.
• Perform a Stress Analysis for sizing.

Shaft Design Guidelines

• Keep shafts short and minimize cantilever designs.
• Hollow shafts have better stiffness/mass ratios, but are more expensive.
• Configure shaft geometry to reduce stress concentrations.
• Remember that gears can produce radial, tangential, and axial loads.
• Be aware of maximum shaft deflection requirements of bearings.
• Shaft natural frequency should be as high as practical.

A shaft may fail by:

・ Excessive lateral deflection, which causes items such as gears to move laterally from their proper location, resulting in incorrect meshing.

・ Torsional deflection, which destroys the precise angular relationship or "timing" between sections of a mechanism.

・ Wear. Wear may take place on bearing surfaces (**JOURNALS**) or other contact areas, such as cams.

・ Fracture. Unless the shaft was grossly under-designed, fracture usually occurs by **FATIGUE CRACKING**.

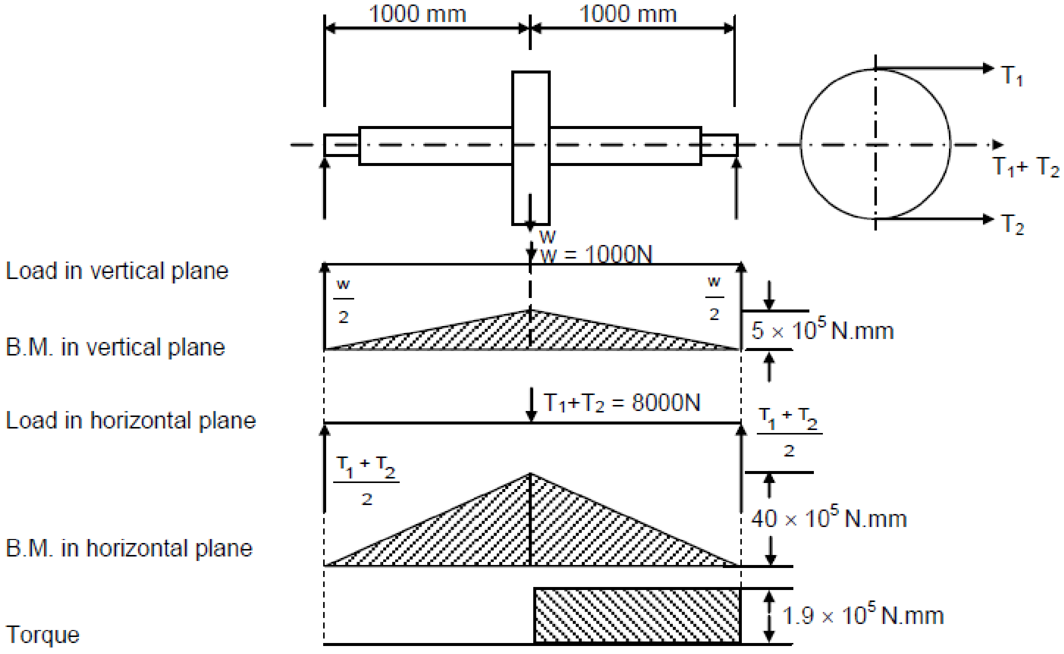
Examples on shaft design:
Example 1:
A shaft carries a 1000 N pulley in the centre of two ball bearings which are 2000 mm apart. The pulley is keyed to the shaft and receives 30 kW of power at 150 rpm. The power is transmitted from the shaft through a flexible coupling just outside the right bearing. The belt derive is horizontal and the sum of the belt tension is 8000 N. Calculate the diameter of the shaft if permissible stress in bending is 80 N/mm2 and in shear it is 45 N/mm2.
Solution:
The belt tensions $T1$ and $T2$ cause horizontal transverse force while weight of the pulley causes vertical transverse force in the middle of the span as shown. The BM diagrams in vertical and horizontal planes and torque diagrams are also shown.
Force in vertical plane = FV = 1000 N
Force in horizontal plane = FH = 8000 N
Both FV and FH act at the mid span. Maximum BM occurs at mid span, assuming that the bearings behave as simple support

$$M_V = \frac{1000 \times 2000}{4} = 0.5 \times 10^6 \text{ Nmm}$$

$$M_H = \frac{8000 \times 2000}{4} = 4 \times 10^6 \text{ Nmm}$$

$$\therefore \text{ Resultant BM, } M = \sqrt{M_V^2 + M_H^2} = 10^6 \sqrt{0.5^2 + 4^2} = 4.031 \times 10^6 \text{ Nmm}$$

$$\text{The shaft torque} \quad M_t = \frac{H}{\omega} = \frac{30 \times 10^3}{2\pi \times \frac{150}{60}} = 1.91 \times 10^3 = 1.9 \times 10^6 \text{ Nmm}$$



$\therefore$ Eq. BM $\quad M_e = \frac{1}{2}[M + \sqrt{M^2 + M_t^2}] = \frac{1}{2}[4.031 + \sqrt{4.031^2 + 1.91^2}] \times 10^6$

$$= 4.25 \times 10^6 \text{ Nmm}$$

and Eq. torque $\quad M_{te} = \sqrt{M^2 + M_t^2} = \sqrt{4.031^2 + 1.91^2} \times 10^6 \text{ Nmm}$

$$= 4.46 \times 10^6 \text{ Nmm}$$

$\therefore$ Permissible bending stress $= 80 \text{ N/mm}^2 = \frac{32 M_e}{\pi d^3}$

$$\therefore \quad d^3 = \frac{32 \times 4.25 \times 10^6}{\pi \times 80} = 0.54 \times 10^6$$

$$\therefore \quad d = 81.5 \text{ mm} \qquad \ldots \text{(a)}$$

Permissible shearing stress $= 45 = \frac{16 M_{te}}{\pi d^3}$

$$\therefore \quad d^3 = \frac{16 \times 4.46 \times 10^6}{\pi \times 45} = 0.5048 \times 10^6$$

$$d = 79.6 \text{ mm} \qquad \ldots \text{(b)}$$

(a) being larger diameter is acceptable, $d = 81.5$ mm

Example 2:
A shaft is supported in ball bearings which are placed 200 mm apart. The shaft carries a straight tooth spur gear of 20° pressure angle at a distance of 50 mm from right hand bearing between the supports. 3.9 kW of power is transmitted by the shaft at 90 rpm. The pitch circle diameter of the gear is 125 mm which receives power from a pinion placed in the same vertical plane above the gear and power is taken off from right hand through a coupling.

Solution:
The shaft is to be made in carbon steel (A1018) for which
τ_p = 126 Mpa
Before proceeding to calculate diameter shaft loading has to be calculated

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\, 90}{60} = 9.425 \text{ rad/s}$$

$$H = 3.9 \times 10^3 \ W = M_t\,\omega \quad M_t \text{ in Nm}$$

$$\therefore \quad M_t = \frac{3.9 \times 10^3}{9.425} = 413.8 \text{ Nm} = 0.414 \times 10^6 \text{ Nmm} \qquad \ldots \text{(i)}$$

The torque *Mt* acts upon the gear at a radius of . 125/2 =62.5 mm. If a tangential force

Pt acts upon the gear at this radius

$$P_t = \frac{M_t}{\frac{d_p}{2}} = \frac{0.414 \times 10^6}{\frac{125}{2}} = 6.621 \times 10^6 \text{ N}$$

This force will act on shaft transversely in horizontal plane (tangential force on gear) at a distance of 50 mm from right hand bearing, which is regarded as simple support along with left hand bearing. The schematic of the shaft is shown in the Figure The bending moment due to *Pt* is calculated below.

Reaction at RH bearing $R_1 = \frac{P_t \times 150}{200} = \frac{6.621 \times 10^3 \times 150}{200} = 4.97 \times 10^3 \text{ N}$

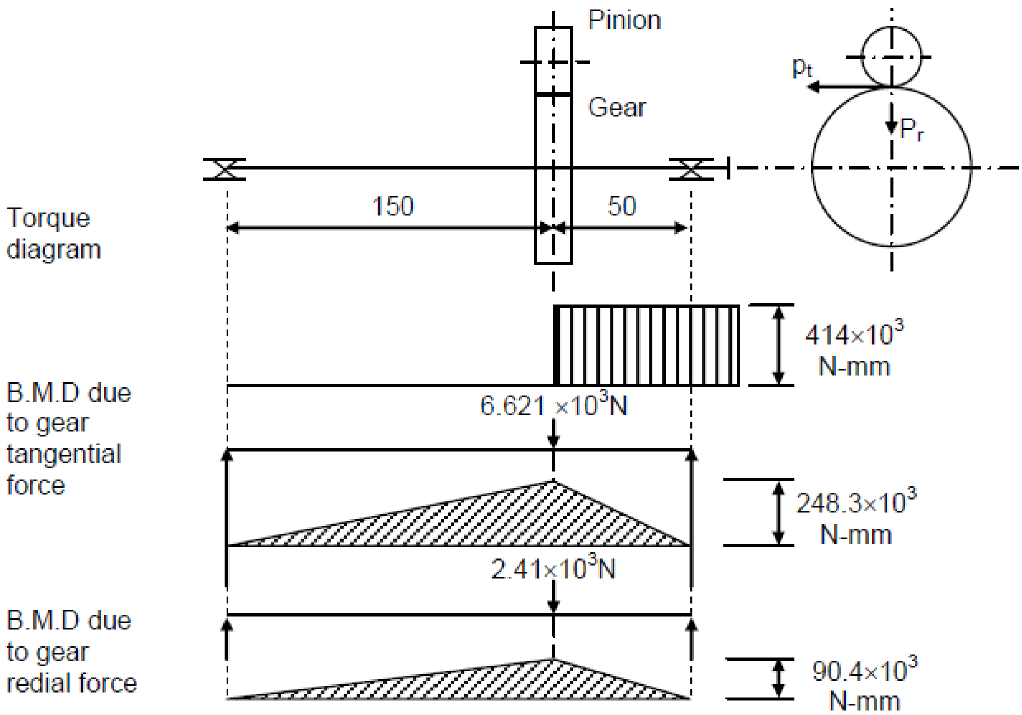
BM at section where gear sits, $M_1 = 4.97 \times 10^3 \times 50 = 248.3 \times 10^3 \text{ Nmm}$ . . . (ii)

The gear will be subjected to a radial force component which will be transmitted to the shaft as transverse load in vertical plane.

The radial force, $P_r = P_t \tan \alpha = 6.621 \times 10^3 \times \tan 20$

$$= 6.621 \times 10^3 \times 0.364$$

or $P_r = 2.41 \times 10^3 \text{ N}$ . . . (iii)



BM due to P_r in vertical plane in gear section

$$M_2 = \frac{P_r \times 150 \times 50}{200} = \frac{2.41 \times 10^3 \times 150 \times 50}{200} = 90.4 \times 10^3 \text{ Nmm}$$

The torque BM in horizontal plane and BM in vertical plane are drawn in

Hence, resultant BM in shaft at section where gear is mounted

$$M = \sqrt{M_1^2 + M_2^2} = 10^3 \sqrt{249.3^2 + 90.4^2} = 264.24 \times 10^3 \text{ Nmm}$$

$$\therefore \quad \text{Eq. BM } M_{eq} = \frac{1}{2}[M + \sqrt{M^2 + (\alpha M_t)^2}]$$

$$M_{eq} = \frac{1}{2}[264.24 + \sqrt{264.24^2 + (0.59 \times 414)^2}] \times 10^3 = 254.7 \times 10^3 \quad \text{. . . (iv)}$$

$$K_m = 2.0, \quad K_t = 1.5$$

$$\therefore \quad M_{teq} = \sqrt{(K_m M)^2 + (K_t M_t)^2}$$

$$= \sqrt{(2 \times 264.24)^2 + (1.5 \times 414)^2} \times 10^3 = \sqrt{0.28 + 0.386} \times 10^6$$

$$= 816 \times 10^3 \text{ Nmm} \quad \text{. . . (v)}$$

The values of M_{eq} and M_{teq} will be used for calculating diameter. With M_{eq} the permissible stress will be fatigue strength in reversible stress cycle, i.e. 65 N/mm^2 (given)

$$\therefore \quad 65 = \frac{32 M_{eq}}{\pi d^3}$$

or

$$d = \left(\frac{32 \times 254.7 \times 10^3 M_{eq}}{\pi \, 65} \right)^{\frac{1}{3}} = 34.17 \text{ mm} \quad \text{. . . (a)}$$

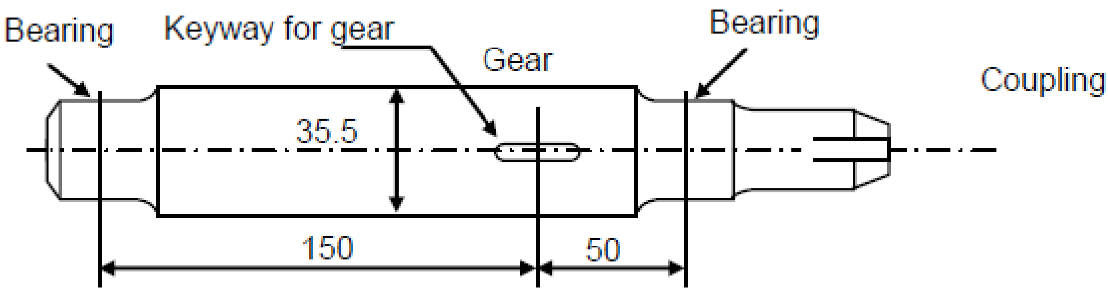
To take care of keyway stress concentration this stress is reduced by 25%.

Hence, $\tau_s = 0.75 \times 126 = 94.5 \text{ N/mm}^2$

$$\therefore \quad 94.5 = \frac{16 M_{eq}}{\pi d^3}$$

or $$d = \left(\frac{16 \times 0.816 \times 10^6}{\pi \times 94.5} \right)^{\frac{1}{3}} = 35.3 \text{ mm} \quad \ldots \text{(b)}$$

Out of the two diameters (a) and (b) the higher value will be chosen. So, d=35.3 mm say 35.5 mm.
The designed shaft will look like the one shown in the following figure.



Constraining Parts on Shafts

- For Torque Transfer
– Keys
– Set screws
– Pins
– Splines
– Tapered fits
– Press or shrink fits
- For Axial Location
– Nut and cotter pins
– Sleeves
– Shoulders
– Ring and groove
– Collar and set screw
– Split hub

Keys and splines:

A key is, usually, a rigid connector between a shaft and the hub or boss of another component such as a pulley, sprocket, lever, flywheel, impellers gear, or cam. It is a piece of steel inserted between the shaft and hub or boss of the pulley to connect these together. Its purpose is to prevent relative rotation between the two parts. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses (σ_{bp} and τ_p).. If a key is to be used, a key-seat must be provided in the shaft and keyway in the hub of the other part. **A keyway** is a slot or recess in the hub or boss of the pulley to accommodate a key. **A key-seat** weakens the shaft by 25% of its allowable stresses. Sharp corner on a key way and key-seat also introduce stress consternation that must be minimised as much as possible. In some cases, tight fit is needed between the key and both the shaft and the hub. In other cases, a tight fit is needed between the key and the shaft, but a loose fit between the key and the hub. The particular type of key specified will depend upon the magnitude of torque transmitted, type of loading (that is, steady, varying, or oscillatory), fit required, limiting shaft stress and cost. Dimension of various types of key have been standardised.

The following types of keys are important from the subject point of view :
(a) Sunk keys, (b) Saddle keys, (c) Tangent keys, (d) Round keys, and (e) Splines.

We shall now discuss the above types of keys, in detail, in the following sections.

Light duty keys:

With light, steady, no-oscillating loads are to be transmitted, the following keys can be used:

Square key: Most common type key where W is equal to one quarter of the shaft diameter.

Flat key: is used where the hub of the is thin. Extra thin flat key is used where both the hollow shaft and the hub are thin.

Woodruff key: It is a light duty key, but has the advantage of being able to align itself readily with the hub, as it is free to rotate within the semicircular key-seat. one of its outstanding advantages is the fact that it cannot possibly be removed (slip out accidentally) unless the shaft and the keyed-on member are separated. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

The shear area for the Woodruff key is the area the top of the shaft. A role of thumb for selecting it is to find one with a width of approximately one-fourth the shaft diameter and a radius approximately one-half the shaft diameter.

Light duty pins:

For light duty or medium operation, it is also possible to use **a taper pin** as a key.

Light duty set screws

These are regarded as light-duty attachments. Sometimes the end of the screw merely bears against the surface of the shaft. In other cases (dog-end or cone-end screws) the end of the screw may enter a drilled hole in the shaft. Unlike bolts and cap screws, which depend on tension to develop a clamping force, the setscrew depends on compression to develop the clamping force. The resistance to axial motion of the collar or hub relative to the shaft is called *holding power*. This holding power, which is really a force resistance, is due to frictional resistance of the contacting portions of the collar and shaft as well as any slight penetration of the setscrew into the shaft.

Medium duty keys:

These keys are similar to the previous category, but tapered.

Plain tapered key: A slope of around 1% is introduced to this type. It is locked in place radialy and axially by wedging action of the key between the hub and the shaft.

Gib head keys: This type of key is tapered 1% and contains a head that projects beyond the keyed members. The head is included to allow for easy removal. The taper is provided to prevent axial displacement by tightly securing the two parts.
A feather key: It is used when it necessary to permit a hub to have axial movement along the shaft and to prevent any rotation between the shaft and the hub. It is either screwed to the shaft with a running fit in the hub or is held in the hub with a running fit in the shaft. As a guide in sizing a feather key, the bearing pressure on its side should not exceed 6.89 MPa

Heavy duty Keys:

There are many types of heavy duty keys such as Nordberg key, the Kennedy key, the Lewis key, and the Barth key.

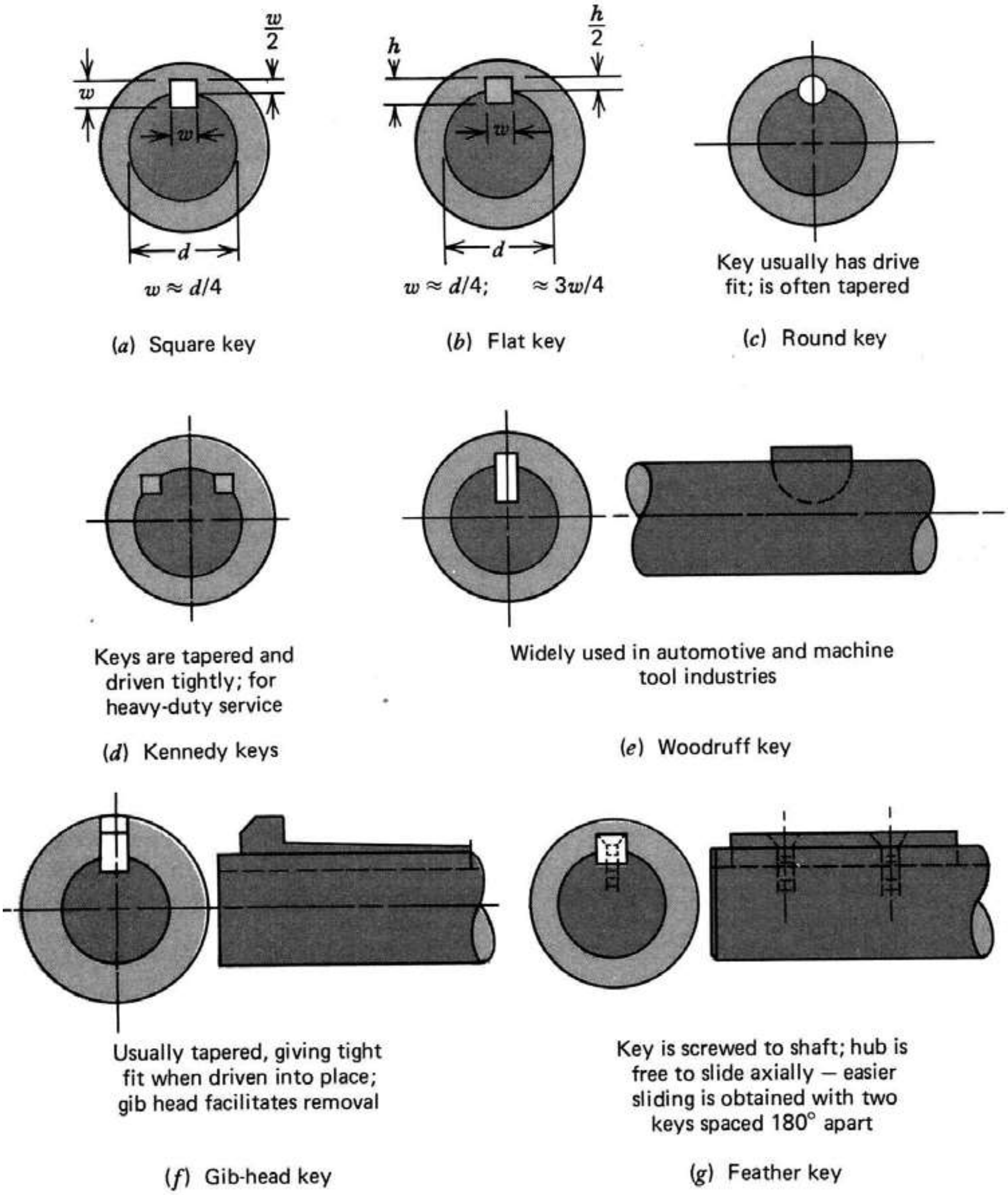


Figure Examples of types of keys in general use

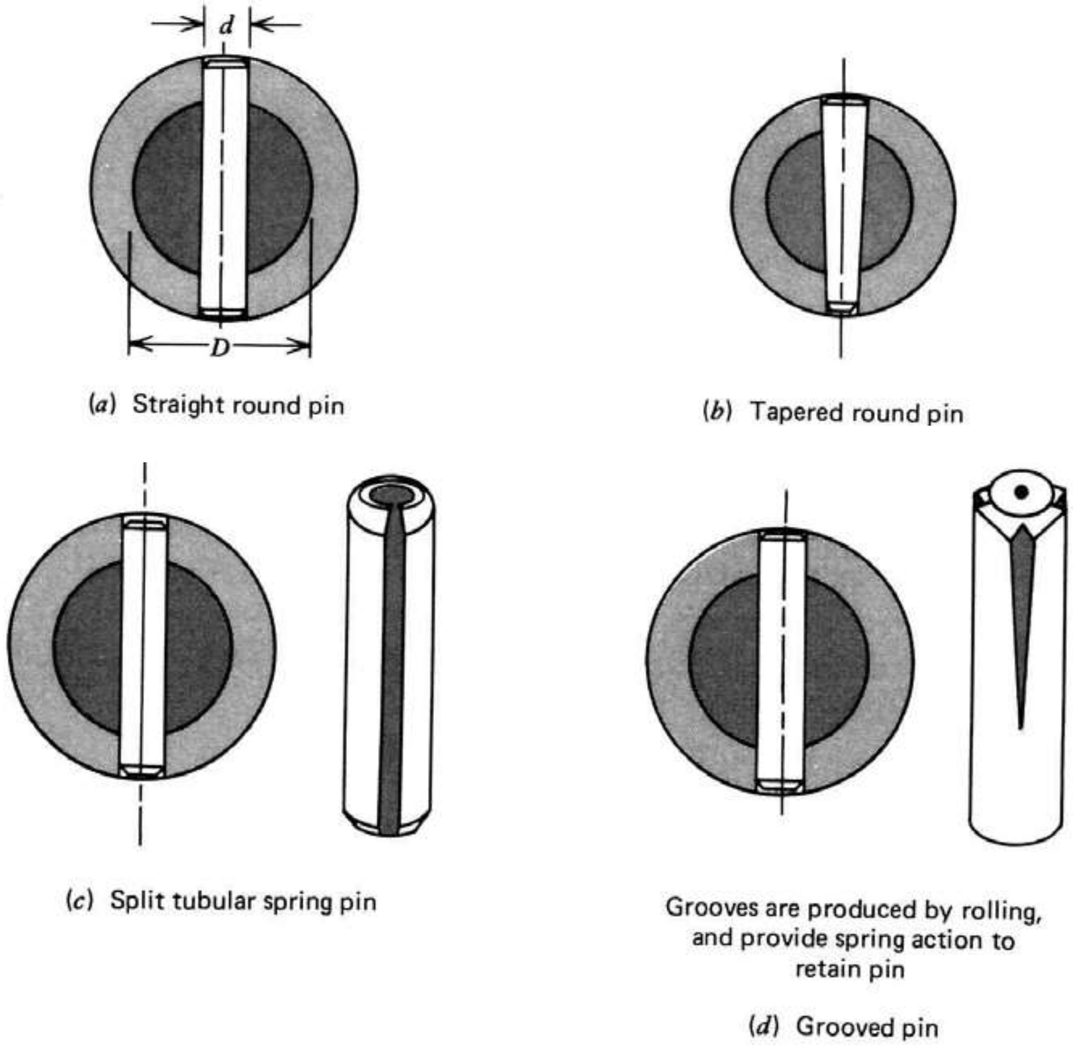


Figure Round pins of various types are sometimes used in place of keys. Pins are generally regarded as suitable only for light duty.

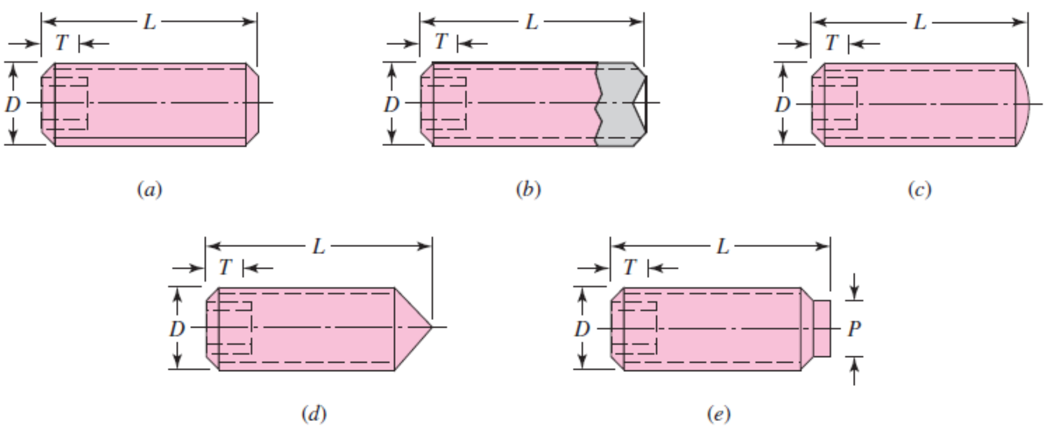


Figure Drawings of different types of setscrews or grub screws. Socket setscrews: (*a*) flat point; (*b*) cup point; (*c*) oval point; (*d*) cone point; (*e*) half-dog point.

Design of a key:

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

(a) Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

(b) Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and crushing stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key,

t = Thickness of key, and

τ_p and σ_{bp}.. = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

F = Area resisting shearing x Shearing stress = $l \times w \times \tau_p$

The force, usually, calculated from the torque transmitted by the shaft,

$F = T / (d/2)$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

F = Area resisting crushing x Crushing stress (σ_{bp}) = $(t/2) \times l \times \sigma_{bp}$

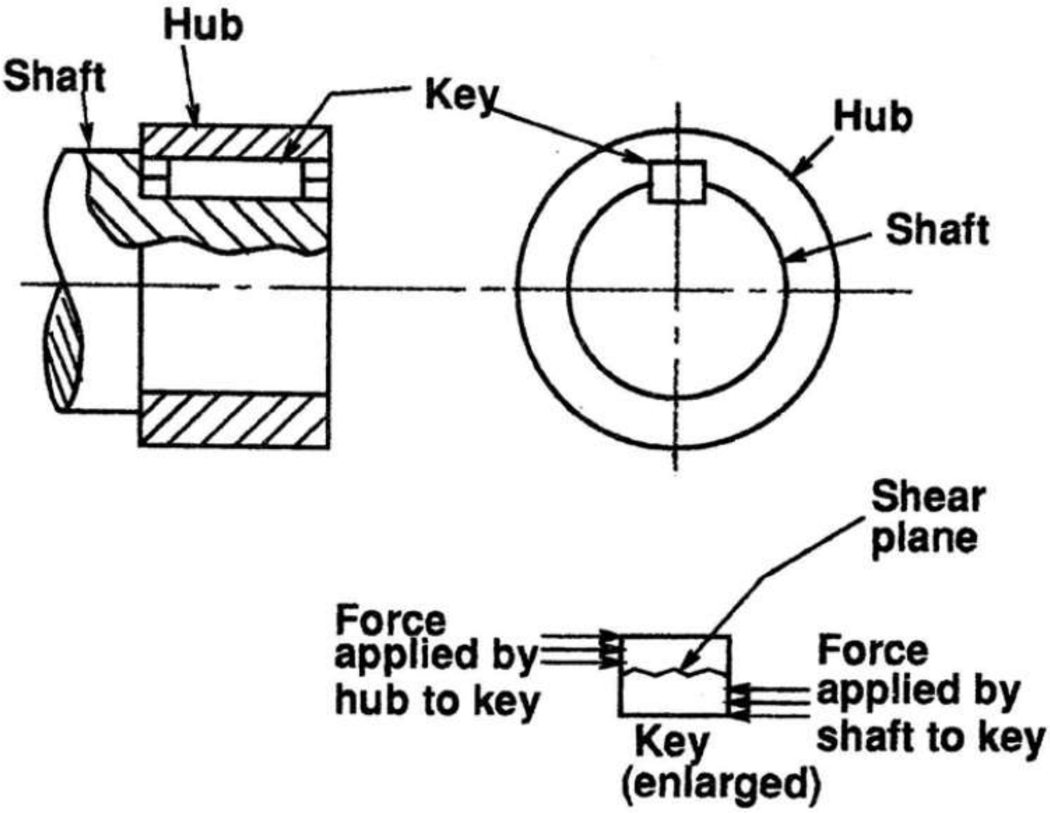
The force, also, calculated from the torque transmitted by the shaft,

$F = T / (d/2)$

In order to find the length of the key to transmit full power of the shaft, the permissible shearing strength of the key is equal to the torsional shear strength of the shaft and the permissible bearing stress of the key is considered the same as the shaft (the same material). The width and the thickness of the key (w and t) are standard values and can be found from standard tables. Two values of the length then can be found and the larger between them will be used.

The maximum length of a key is limited by the hub length of the attached element, and should generally not exceed about 1.5 times the shaft diameter to avoid excessive twisting with the angular deflection of the shaft. Multiple keys may be used as necessary to carry greater loads, typically oriented at 90° from one another. Excessive safety factors should be avoided in key design, since it is desirable in an overload situation for the key to fail, rather than more costly components.

Stock key material is typically made from low carbon cold-rolled steel, and is manufactured such that its dimensions never exceed the nominal dimension. This allows standard cutter sizes to be used for the key-seats. A setscrew is sometimes used along with a key to hold the hub axially, and to minimize rotational backlash when the shaft rotates in both directions.



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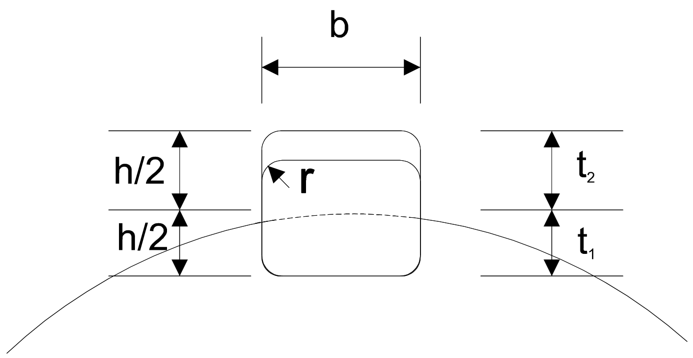
Dimensions of Parallel keys from BS 4235:1972



Shaft dia. mm		Section	Width b Tolerance for class fit										
			Free		Normal		close	Shaft t_1		Hub t_2		Rad. r	
over	incl.	b×h	Shaft	Hub	Shaft	Hub	Shaft Hub	nom	tol	nom	tol	max.	min
22	30	8×7	+0.036	+0.098	0.0	+0.018	-0.01	4		3.3		0.25	0.16
30	38	10×8	0.0	+0.040	-0.036	-0.018	-0.051	5		3.3		0.40	0.25
38	44	12×8						5	+0.2	3.3	+0.2	0.40	0.25
44	50	14×9	+0.043	+0.120	0.0	+0.021	-0.018	5.5	0.0	3.8	0.0	0.40	0.25
50	58	16×10	0.0	+0.050	-0.043	-0.021	-0.061	6		4.3		0.40	0.25
58	65	18×11						7		4.4		0.40	0.25

Table: Proportions of Standard Parallel, Tapered and Gib Head Key according to IS : 2292 and 2293-1974

Shaft Diameter (mm) up-to and Including	**Key Cross-section**		**Shaft Diameter (mm) up-to and Including**	**Key Cross-section**	
	Width (mm)	**Thickness (mm)**		**Width (mm)**	**Thickness (mm)**
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

Woodruff key:

The Woodruff key, as shown in following figure, is of general usefulness, especially when a wheel is to be positioned against a shaft shoulder, since the keyslot need not be machined into the shoulder stress concentration region. The use of the Woodruff key also yields better concentricity after assembly of the wheel and shaft. This is especially important at high speeds, as, for example, with a turbine wheel and shaft. Woodruff keys are particularly useful in smaller shafts where their deeper penetration helps prevent key rolling. Dimensions for some standard Woodruff key sizes can be found in the next table, gives the shaft diameters for which the different keyseat widths are suitable.

Table Dimensions of woodruff key

Key number	Nominal key size, $W \times B$	Actual length, F	Height of key, C	Shaft keyseat depth	Hub keyseat depth
202	1/16 × 1/4	0.248	0.104	0.0728	0.0372
204	1/16 × 1/2	0.491	0.200	0.1668	0.0372
406	1/8 × 3/4	0.740	0.310	0.2455	0.0685
608	3/16 × 1	0.992	0.435	0.3393	0.0997
810	1/4 × 1 1/4	1.240	0.544	0.4170	0.1310
1210	3/8 × 1 1/4	1.240	0.544	0.3545	0.1935
1628	1/2 × 3 1/2	2.880	0.935	0.6830	0.2560
2428	3/4 × 3 1/2	2.880	0.935	0.5580	0.3810

Source: Reprinted from ASME B17.2-1967, by permission of the American Society of Mechanical Engineers.

Note: All dimensions are given in inches.

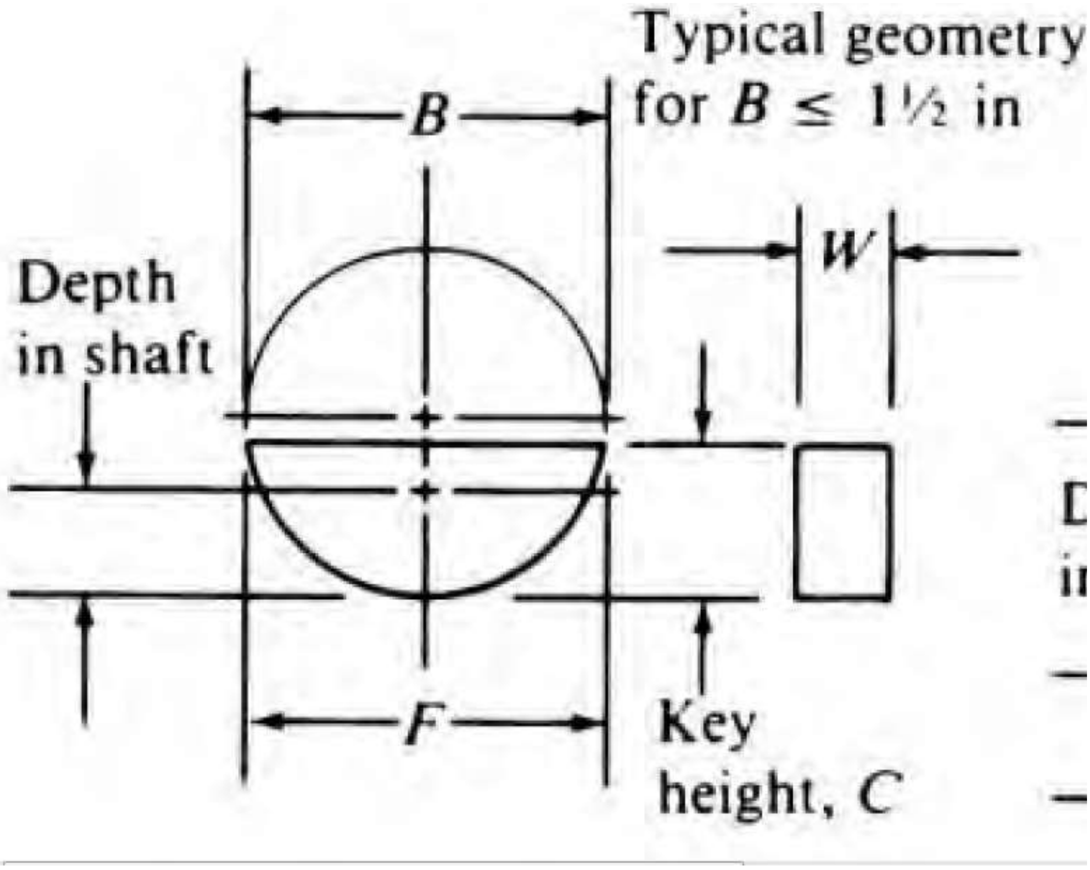


Figure Dimensions of woodruff key

Splines

When a shaft is required to carry torque beyond that obtainable with keys (or when the is frequently reversed), one solution is to spline the shaft and the hub of the connected member. In this way, the keys are made integral with the shaft which fit in the keyways broached in the hub. Such shafts are known as **splined shafts**. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive.

Splines are essentially axial grooves or recesses which are machined into the shaft, very like a series of keyways. Splines are an integral part of the shaft (opposite to keys, which are loose parts). Corresponding grooves are cut (**BROACHED**) into the bore of the hub so that the shaft/hub assembly forms a series of interlocking projections. The resulting connection is stronger than a keyed joint and is used in heavy-duty applications. Spline profiles may be square, involute or triangular. Splines are often designed to allow axial movement of a gear or hub whilst continuing to transmit torque. One particular application is in a multi-speed gearbox. For axial sliding to occur satisfactorily, the bearing pressure on the faces of the spline must be low and good lubrication must be provided.

Two types of **splines** have been standardised, the **ASA involute spline** with five different angles and the **SAE straight spline** with four different number of splines.

Common designs use spline lengths of 0.75 D to 1.25 D, where D is the pitch diameter of the spline. When these standard lengths are used, the shear strength of the splines will exceed that of the shaft from which they are made.
Involute splines are typically made with pressure angles of 30°, 37.5°, or 45°.

Standard Diametral Pitches. The following are the 17 standard diametral pitches in common use:
2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 24, 32, 40, 48, 64, 80, 128

Length of Splines. Common designs use spline lengths from 0.75D to 1.25D, where *D* is the pitch diameter of the spline. If these standards are used, the shear strength of the splines will exceed that of the shaft on which they are machined.

Standard Modules. There are 15 standard modules:
0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.50, 3, 4, 5, 6, 8, 10.

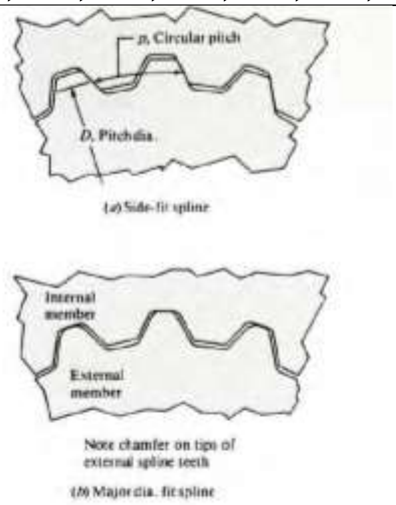
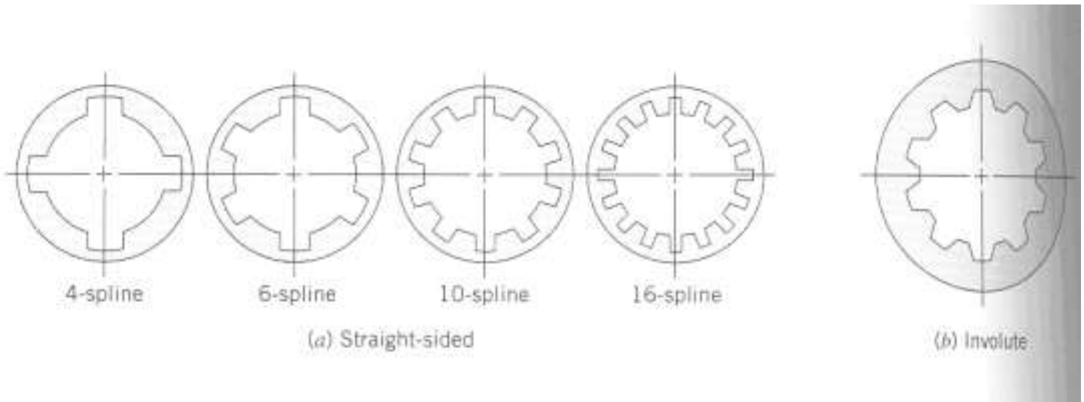
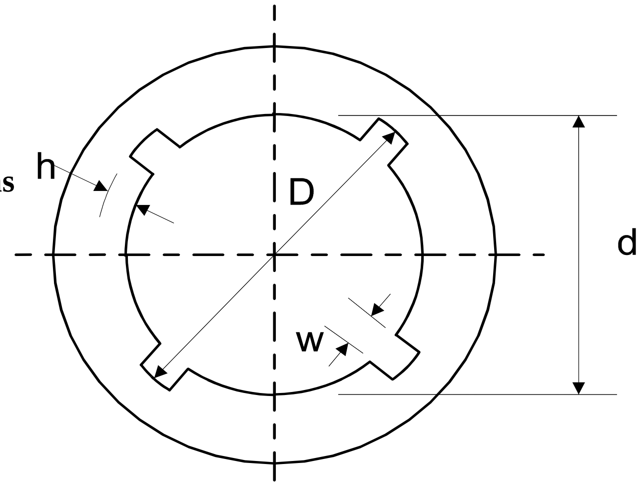


Figure: 30° involute spline



Common types of splines.

SAE Straight Splines, Nominal dimensions



No. of splines	Permanent Fit		To slide when not under load		To slide when under load		All Fits
	h	d	h	d	h	d	w
4	0.075 D	0.850 D	0.125 D	0. 750 D	-	-	0.241 D
6	0.050 D	0.900 D	0.075 D	0.850 D	0.100 D	0.800 D	0.250 D
10	0.045 D	0.910 D	0.070 D	0.860 D	0.095 D	0.810 D	0.156 D
16	0.045D	0.910 D	0.070 D	0.860 D	0.095 D	0.810 D	0.098 D

Bearings

Bearings may be classified in a number of different ways. However, for our purposes, it is sufficient to use two groups:

- **Sliding contact**
- **Rolling contact**

Before moving on to consider these two groups, it is worth pointing out the necessary conditions for stable and adequately constrained mounting of a shaft. Whilst being free to rotate, the whole shaft must generally be constrained against **RADIAL** movement and against **AXIAL** movement. In general, adequate radial constraint requires two bearings, relatively widely spaced along the length of the shaft. Only in the case of **very** short shafts should the use of only one bearing be considered. Again in general, the shaft will need axial restraint in two directions. Sometimes both axial restraints are applied by one bearing; in other cases, each of two bearings may provide restraint in one axial direction. Long shafts usually require more than two bearings, especially if lateral rigidity (small deflection under load) is required. Especially for long shafts, questions of axial expansion due to heating must be considered. Potential changes of the length of either the housing or the shaft (or both) due to heating while in use usually require both axial constraints to be provided by the same bearing.

Sliding contact bearings

The design of sliding bearing is mainly based on a large number of design charts and depends the type of the bearings. Many types of bearings are known. Only introduction will be presented here, and full detailed design will be left for the future when the engineer become in charge of such task.

Sliding bearings (also called *plain bearings*) are of two types: (1) *journal* or *sleeve bearings*, which are cylindrical and support radial loads (those perpendicular to the shaft axis); and (2) *thrust bearings*, which are generally flat and, in the case of a rotating shaft, support loads in the direction of the shaft axis.

The simplest type of these bearings is known as a **JOURNAL** or **PLAIN** or **SLEEVE BEARING**. A bearing of this type locates a shaft **RADIALLY**. There is no provision for axial location.

Materials for plain bearings

Where rubbing contact occurs between two machine parts, it is usual to make the parts of dissimilar materials. In the case of journal bearings, the shaft to be supported is almost always made from carbon steel, so bearings are seldom made of steel. Frequently used bearing materials are:

- Bronze, usually in the form of a bush.
- White metal, a tin/antimony/copper alloy which is often bonded to a steel shell.
- Copper/lead/indium, often used for automotive engine bearings, usually bonded to a steel shell.
- Cast iron, with a shaft running directly in a machined bore in the cast component.
- Various **plastics such as PTFE, nylon, delrin.**

Lubrication of plain bearings

Plain bearings are usually lubricated by grease or oil, supplied by an oil drip lubricator or a ring oiler or by periodic application of an oil-can. Alternatively, it is possible to design a plain bearing as an air- or gas-lubricated bearing, which requires a constant supply of gas under pressure. Gas-lubricated bearings are beyond the scope of these notes.

Characteristics of plain bearings

- Friction is higher than for rolling-contact bearings. Starting friction of plain bearings is significantly higher than running friction.
- Well-designed plain bearings can have an extremely long life. However, they can fail without warning.
- Plain bearings run more quietly than rolling-contact bearings.

Lubrication Theory

Types of Lubrication: Lubrication is commonly classified according to the degree with which the lubricant separates the sliding surfaces. The following figure illustrates three basic cases.

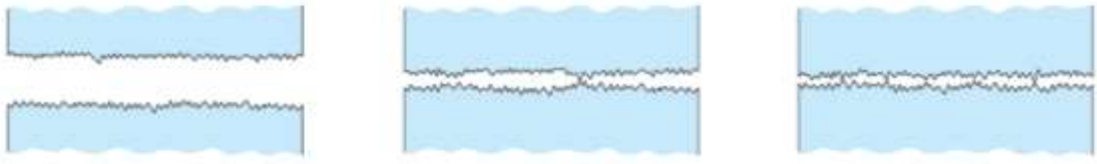


FIGURE : Three basic types of lubrication. The surfaces are highly magnified.

1. In *hydrodynamic lubrication* the surfaces are completely separated by the lubricant film. The load tending to bring the surfaces together is supported entirely by fluid pressure generated by relative motion of the surfaces (as journal rotation). Surface wear does not occur, and friction losses originate only within the lubricant film. Typical film thicknesses at the thinnest point (designated h_0) are 0.008 to 0.020 mm (0.0003 to 0.0008 in.). Typical values of coefficient of friction (f) are 0.002 to 0.010.
2. In *mixed-film lubrication* the surface peaks are intermittently in contact, and there is partial hydrodynamic support. With proper design, surface wear can be mild. Coefficients of friction commonly range from 0.004 to 0.10.
3. In *boundary lubrication* surface contact is continuous and extensive, but the lubricant is continuously "smeared" over the surfaces and provides a continuously renewed adsorbed surface film that reduces friction and wear. Typical values of f are 0.05 to 0.20.

Complete surface separation (as in the above Figure *a*) can also be achieved by *hydrostatic* lubrication. A highly pressurized fluid such as air, oil, or water is introduced into the load-bearing area. Since the fluid is pressurized by external means, full surface separation can be obtained whether or not there is relative motion between the surfaces. The principal advantage is extremely low friction at all times, including during starting and low-speed operation. Disadvantages are the cost, complication, and bulk of the external source of fluid pressurization. Hydrostatic lubrication is used only for specialized applications.

Whenever a solid surface moves over another, it must overcome a resistive, opposing force known as *solid friction*. The first stage of solid friction, known as *static friction*, is the frictional resistance that must be overcome to initiate movement of a body at rest. The second stage of frictional resistance, known as *kinetic friction*, is the resistive force of a body in motion as it slides or rolls over another solid body. It is usually smaller in magnitude than static friction. Although friction varies according to applied load and solid surface roughness, it is unaffected by speed of motion and apparent contact surface area. When viewed under a microscope a solid surface will appear rough with many *asperities* (peaks and valleys). When two solid surfaces interact without a lubricating medium, full metal-to-metal contact takes place in which the asperity peaks of one solid interferes with asperity peaks of the other solid. When any movement is initiated the asperities collide causing a rapid increase in heat and the metal peaks to adhere and weld to one another. If the force of motion is great enough the peaks will plow through each other's surface and the welded areas will shear causing surface degradation, or wear. In extreme cases, the resistance of the welded solid surfaces could be greater than the motive force causing mechanical seizure to take place. Some mechanical systems designs, such as brakes, are designed to take advantage of friction. For other systems, such as bearings, this metal-to-metal contact state and level of wear is usually undesirable. To combat this level of solid friction, heat, wear, and consumed power, a suitable lubricating fluid or fluid film must be introduced as an intermediary between the two solid surfaces. Although lubricants themselves are not frictionless, the molecular resistive force of a gas or fluid in motion known as *fluid friction* is significantly less than *solid friction*. The level of fluid friction is dependent on the lubricant's *Viscosity*

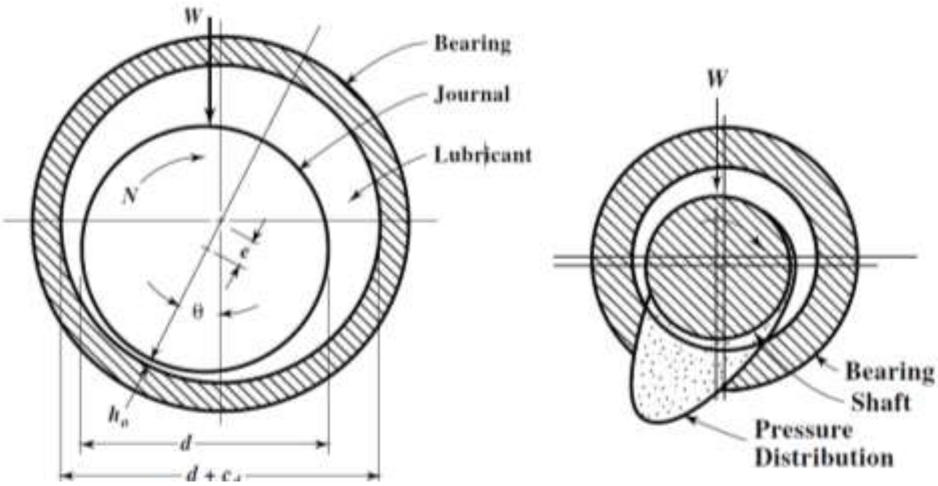


Figure: Basic components of a journal bearing. and Typical pressure profile of journal bearing.

The following is a list of important factors to be taken into account when designing a bearing for hydrodynamic lubrication.

1. The minimum oil film thickness must be sufficient to ensure thick-film lubrication.

2. Friction should be as low as possible, consistent with adequate oil film thickness. Try to keep in the "optimum zone".

3. Be sure that an *adequate supply* of *clean* and *sufficiently cooled* oil is always available at the bearing inlet. This may require forced feeding, special cooling provisions, or both.

4. Be sure that the maximum oil temperature is acceptable (generally below 93° to 121°C or 200° to 250°F).

5. Be sure that oil admitted to the bearing gets distributed over its full length. This may require grooves in the bearing. If so, they should be kept away from highly loaded areas.

6. Select a suitable bearing material to provide sufficient strength at operating temperatures, sufficient comformability and embeddability, and adequate corrosion resistance.

7. Check the overall design for shaft misalignment and deflection. If these are excessive, even a properly designed bearing will give trouble.

8. Check the bearing loads and elapsed times during start-up and shutdown. Bearing pressures during these periods should preferably be under 2 MPa, or 300 psi. If there are extended time periods of low-speed operations, thin-film lubrication requirements must be considered.

9. Be sure that the design is satisfactory for all reasonably anticipated combinations of clearance and oil viscosity. The operating clearance will be influenced by thermal expansion and by eventual wear. Oil temperature and therefore viscosity is influenced by thermal factors (ambient air temperature, air circulation, etc.), and by possible changes in the oil with time. Furthermore, the user may put in a lighter or heavier grade of oil than the one specified

Thrust Bearings

As the name implies, thrust bearings are used either to absorb axial shaft loads or to position shafts axially. Brief descriptions of the normal designs for these bearings follow with approximate design methods for each. The generally accepted load ranges for these types of bearings are given in the following Table and the schematic configurations are shown in the following Figure.

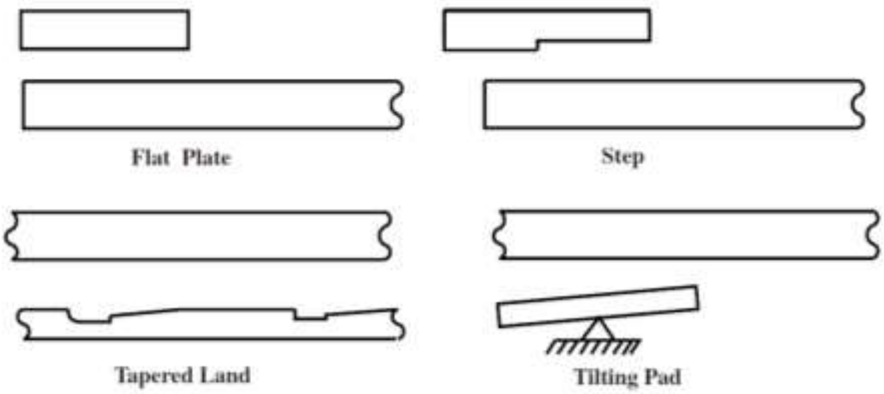


Figure: Types of thrust bearings.

Table : Thrust Bearing Loads

Type	Normal Unit Loads, [Lb per Sq. In.]	Maximum Unit Loads, [Lb per Sq. In.]
Parallel surface	<75	<150
Step	200	500
Tapered land	200	500
Tilting pad	200	500

The parallel or flat plate thrust bearing is probably the most frequently used type. It is the simplest and lowest in cost of those considered; however, it is also the least capable of absorbing load, as can be seen from Table 1. It is most generally used as a positioning device where loads are either light or occasional.

The step bearing, like the parallel plate, is also a relatively simple design. This type of bearing will accept the normal range of thrust loads and lends itself to low-cost, high-volume production. However, this type of bearing becomes sensitive to alignment as its size increases.

The tapered land thrust bearing, as shown in Table 1, is capable of high load capacity. Where the step bearing is generally used for small sizes, the tapered land

type can be used in larger sizes. However, it is more costly to manufacture and does require good alignment as size is increased.
The tilting pad or Kingsbury thrust bearing (as it is commonly referred to) is also capable of high thrust capacity. Because of its construction it is more costly, but it has the inherent advantage of being able to absorb significant amounts of misalignment.

Rolling contact bearings

These bearings use elements such as balls or rollers to avoid sliding contact. The usual practice is to provide two more-or-less cylindrical housings or **RACES**, the inner one of which fits onto the shaft and rotates with it, while the outer race fits into the fixed or non-rotating component. The rolling members (balls or rollers) run freely between the two races. In practice, it is impossible to achieve pure rolling and there will always be some sliding movement within the bearing. Despite this, such bearings run very freely and are often referred to as **ANTI-FRICTION BEARINGS**
Characteristics of rolling contact bearings:

- They have very low friction, particularly when starting.
- They require larger radial sizes than plain bearings but need shorter axial length.
- They are generally more costly than plain bearings.
- They have a finite life. The rolling elements and races eventually fail by flaking of the contact surfaces, a form of fatigue failure.
- Rolling contact bearings are not as quiet in operation as plain bearings. As progressive surface flaking occurs, the bearings become increasingly noisy in operation and provide ample warning of impending failure. Compare with plain bearings which can fail very rapidly and almost without warning.

Bearing types:
Bearings are manufactured to take pure radial loads, pure thrust loads, or a combination of the two kinds of loads. Rolling-element bearings are either *ball bearings* or *roller bearings.* In general, ball bearings are capable of higher speeds, and roller bearings can carry greater loads. Most rolling-element bearings can be classed in one of three categories:
(1) *radial* for carrying loads that are primarily radial; (2) *thrust*, or axial-contact for carrying loads that are primarily axial; and (3) *angular-contact* for carrying combined axial and radial loads.
The nomenclature of a ball bearing is illustrated in following Figure, which also shows the four essential parts of a bearing. These are the outer ring, the inner ring, the balls or rolling elements, and the separator. In low-priced bearings, the separator is sometimes omitted, but it has the important function of separating the elements so that rubbing contact will not occur. In this section we include a selection from the many types of standardized bearings that are manufactured. Most bearing manufacturers provide engineering manuals and brochures containing lavish descriptions of the various types available. In the small space available here, only a meager outline of some of the most common types can be given. So you should include a survey of bearing manufacturers' literature in your studies of this section. Some of the various types of standardized bearings that are manufactured are shown in the following figure. The single-row deep-groove bearing, for example, will take radial load as well as some thrust load. The balls are inserted into the grooves by moving the inner ring to an eccentric position. The balls are separated after loading, and the separator is then inserted.

As a second example, the following figure shows the nomenclature of a tapered roller bearing, and the point *G* through which radial and axial components of load act. The four components of a tapered roller bearing assembly are the:

• Cone (inner ring)
• Cup (outer ring)
• Tapered rollers
• Cage (spacer-retainer)

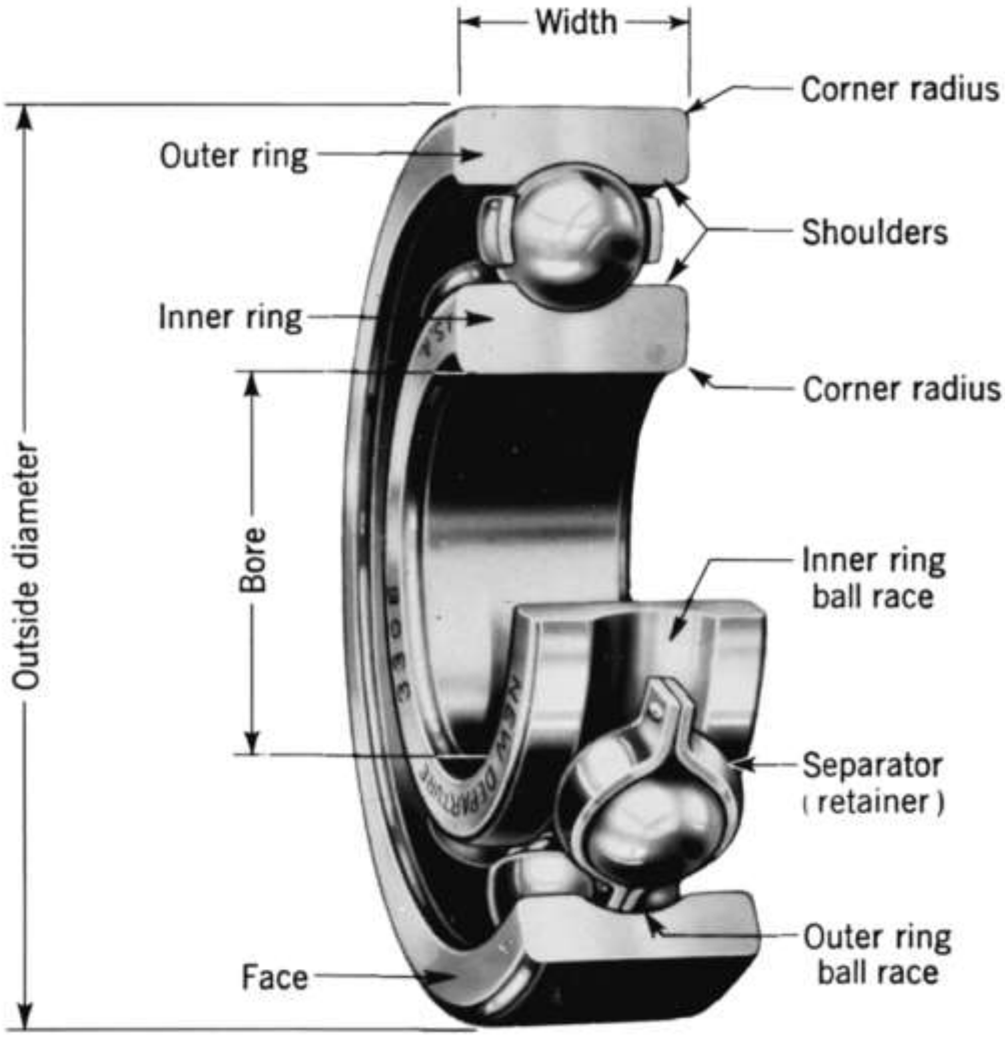


Figure : Nomenclature of a ball bearing. *(General Motors Corp. Used with permission, GM Media Archives.)*

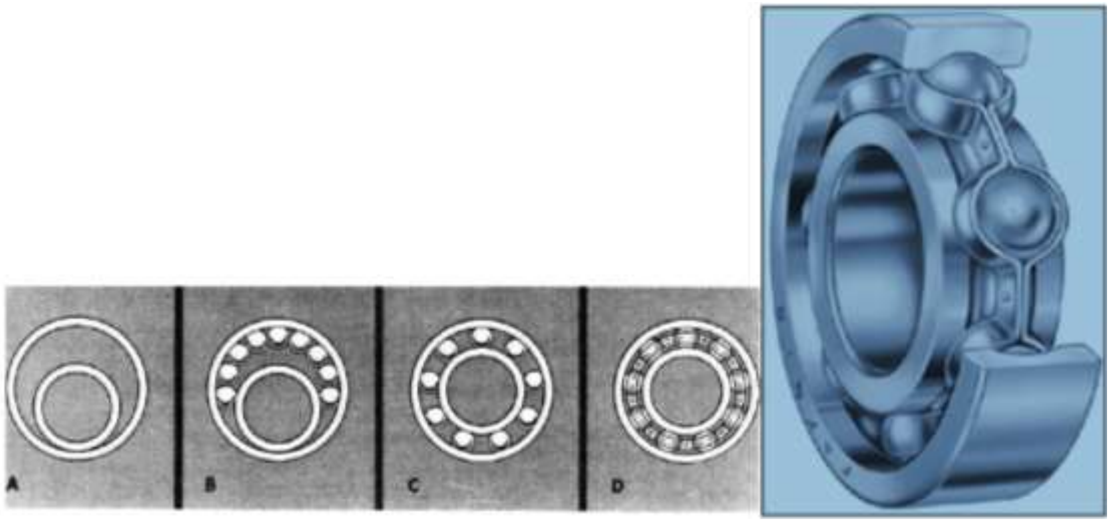


Figure Steps in the assembly of the **CONRAD** or **DEEP GROOVE** type ball bearing.

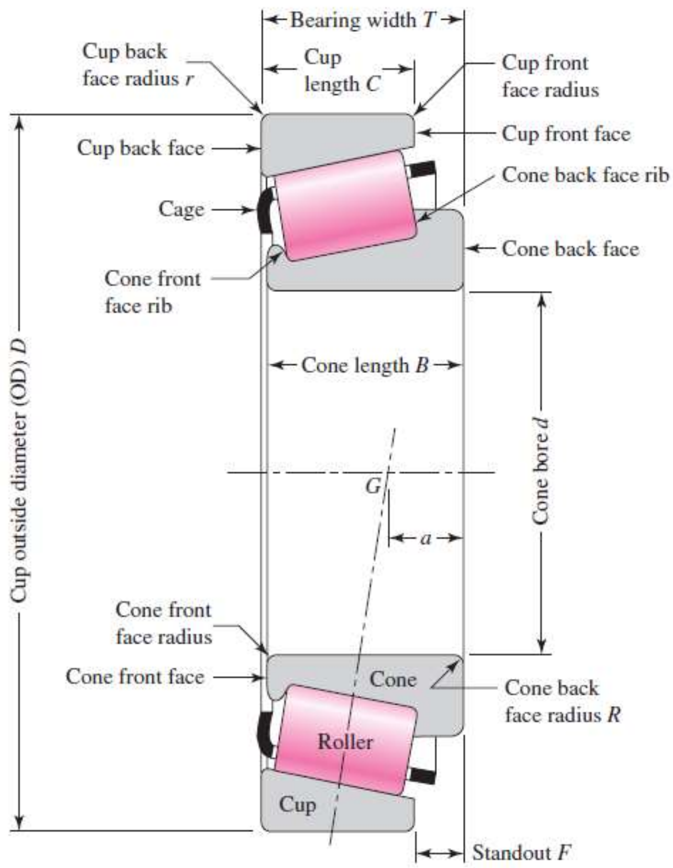


Figure : Nomenclature of a tapered roller bearing. Point G is the location of the effective load center; use this point to estimate the radial bearing load. *(Courtesy of The Timken Company.)*

Ball bearings:
Deep groove ball bearings

In ball bearings in general, the rolling elements are spheres of high quality, hardened and polished alloy steel, rolling in hardened alloy steel **INNER** and **OUTER RINGS** or **RACES**.

An alternative to **DEEP GROOVE BALL BEARINGS** is the **FILLING NOTCH BALL BEARING**, in which small "notches" are cut out of one side of the inner and outer races. When the notches are aligned, the required number of balls may be slipped into the raceways before being spaced by the cage, as for the deep groove bearing. However, the notches create a slight discontinuity in the path of the balls, tending to decrease the life of the bearing. Since in deep-groove bearings the balls run in shallow grooves in the races these bearings are able to support some axial loading as well as radial. Where greater axial thrust is expected, angular contact ball bearings are preferred and, where the expected load is entirely axial, a ball thrust bearing, may be used.

Angular contact ball bearings

Angular contact ball bearings, capable of carrying a combination of axial and radial loading. Note how one side of the groove in both the inner and outer races provides more contact with the balls than the other side. The bearing must therefore be mounted in the correct orientation to carry the axial load.

A **BALL THRUST BEARING**; *and:* A **ROLLER THRUST BEARING**: Both bearings are used in conjunction with **THRUST WASHERS**, one on top, one underneath, made from hardened and polished steel.

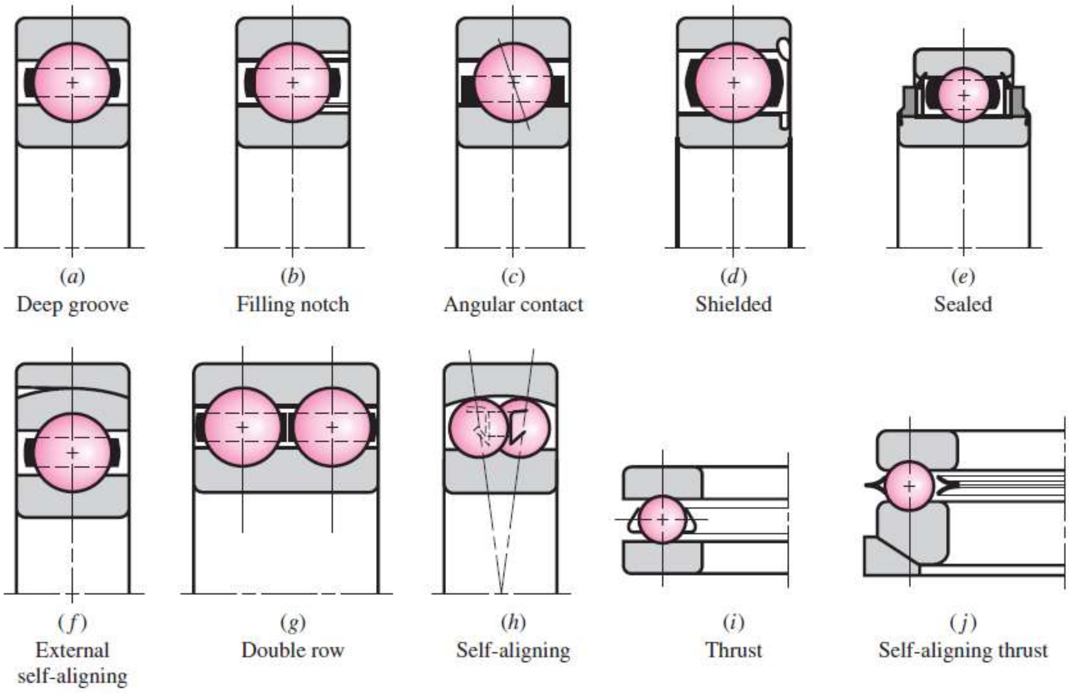


Figure Various types of ball bearings.

Roller bearings:

Cylindrical roller bearings

Whereas ball bearings have theoretical "point" contact between the balls and races, roller bearings have "line" contact and so can carry heavier loads. In practice, of course, both balls and rollers deform under load and contact between the rolling element and the races changes from a point or line to a small area.

In general, cylindrical roller bearings do not provide axial restraint – the rollers in their cage are free to slide axially along the outer race. Whilst this feature may be useful to allow for the change of shaft length due to heating, these bearings cannot be used unless there is provision elsewhere to provide axial location and, if necessary, carry axial loading or thrust.

Some types of cylindrical roller bearings do provide axial restraint in one direction by means of a shoulder in the outer race. Note that any axial or thrust loading carried by a roller bearing must be resisted by the ends of the rollers contacting the shoulders in the inner and outer races. Since this must result in sliding motion rather than rolling, axial loading must be kept low to avoid scoring, overheating and seizure.

Tapered roller bearings:

Tapered roller bearings have very high radial-load capacity and high axial-load capacity in one direction. For this reason, tapered roller bearings are often used in pairs, mounted in opposite directions, to cater for high radial loads and axial loads in either direction. Tapered roller bearings are often used in the wheel bearings of cars and heavy trucks. Note that each roller is actually a **FRUSTRUM** of a cone so that true rolling contact is maintained over the whole length of the roller. A double row tapered roller bearing, used to increase the load-carrying capacity of the bearing and to provide axial restraint in two directions.

Roller thrust bearings

Roller thrust bearings are again tapered rollers to maintain true rolling over the length of the roller. Roller thrust bearings have a greater load capacity than ball thrust bearings.

Needle roller bearings

One of the main advantages of needle roller bearings is their small diameter, so they can be fitted into small spaces. They usually do not have a separate inner race, running directly onto the shaft, which must be suitably hard and with a precision ground surface. When these bearings fail, they often damage the shaft surface and the shaft needs to be replaced. Compare with ball and roller bearings, which have an inner race so that bearing damage is normally confined to the bearing races and the shaft is undamaged.

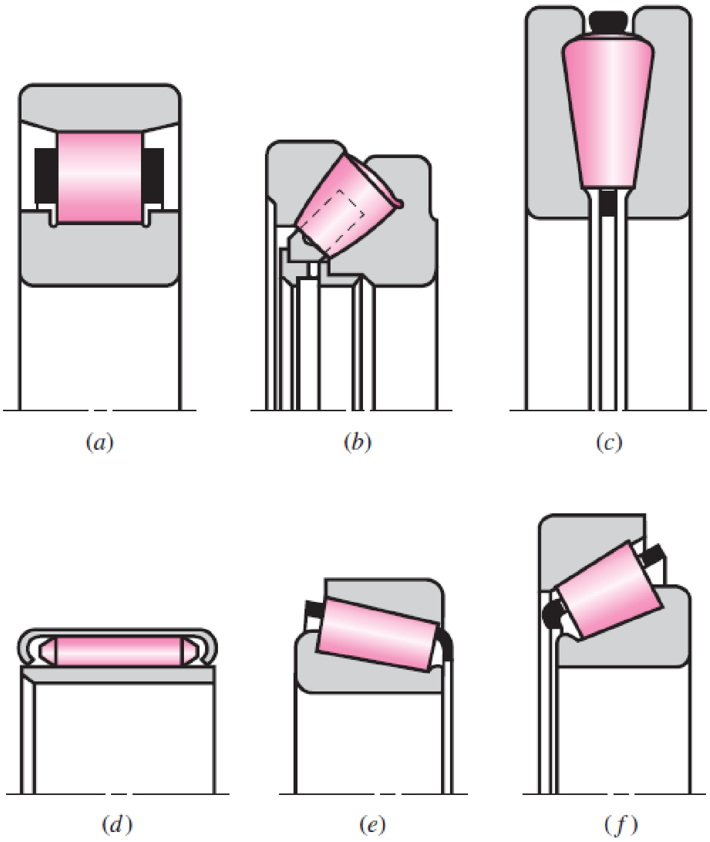


Figure Types of roller bearings: (*a*) straight roller; (*b*) spherical roller, thrust; (*c*) tapered roller, thrust; (*d*) needle; (*e*) tapered roller; (*f*) steep-angle tapered roller. *(Courtesy of The Timken Company.)*.

Bearings Standard dimensions:

The **ABMA** has established standard boundary dimensions for bearings, which define the bearing bore, the outside diameter (OD), the width, and the fillet sizes on the shaft and housing shoulders. The basic plan covers all ball and straight roller bearings in the metric sizes. The plan is quite flexible in that, for a given bore, there is an assortment of widths and outside diameters. Furthermore, the outside diameters selected are such that, for a particular outside diameter, one can usually find a variety of bearings having different bores and widths.

This basic **ABMA** plan is illustrated in following figure. The bearings are identified by a two-digit number called the ***dimension-series code***. The first number in the code is from the ***width series***, 0, 1, 2, 3, 4, 5, and 6. The second number is from the ***diameter series*** (outside), 8, 9, 0, 1, 2, 3, and 4. The Figure shows the variety of bearings that may be obtained with a particular bore. Since the dimension-series code does not reveal the dimensions directly, it is necessary to resort to tabulations. The housing and shaft shoulder diameters listed in the tables should be used whenever possible to secure adequate support for the bearing and to resist the maximum thrust loads.

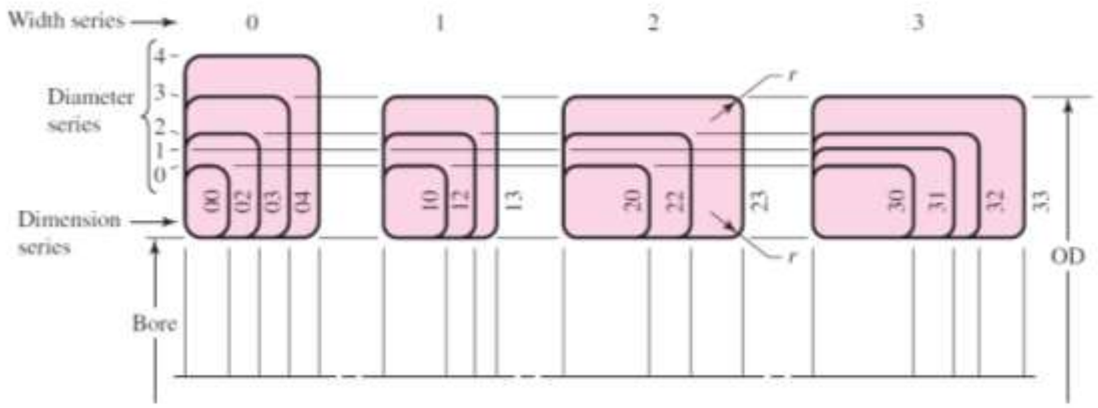


Figure The basic ABMA plan for boundary dimensions. These apply to ball bearings, straight roller bearings, and spherical roller bearings, but not to inch-series ball bearings or tapered roller bearings. The contour of the corner is not specified. It may be rounded or chamfered, but it must be small enough to clear the fillet radius specified in the standards..

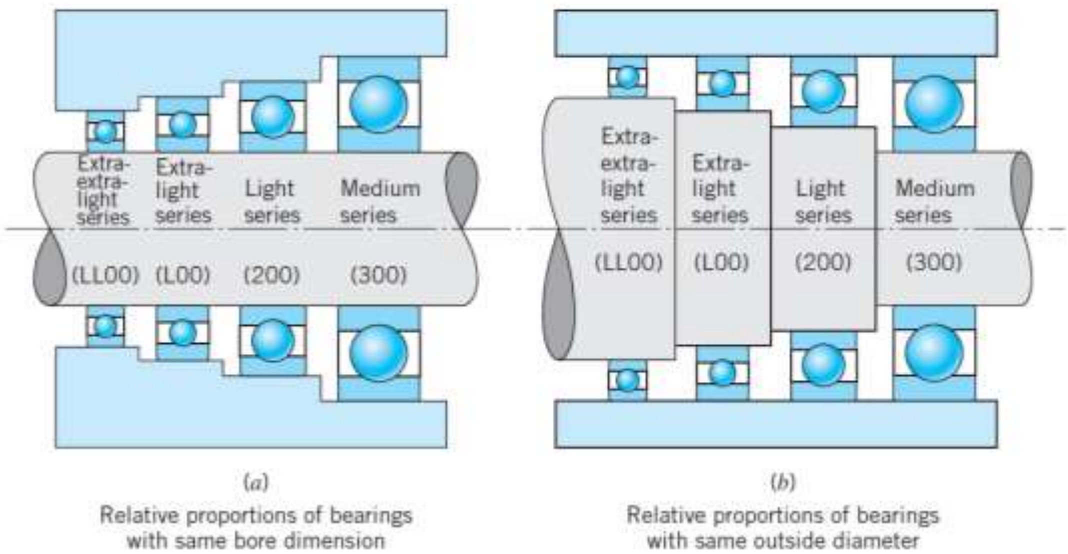


Figure Relative proportions of bearings of different series.

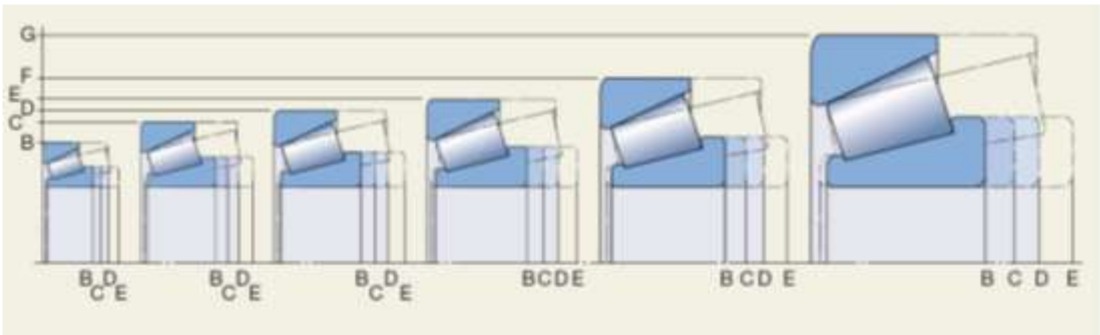


Figure The basic ISO plan for boundary dimensions. These apply to tapered roller bearings.

Fitting of Rolling-Element Bearings

Normal practice is to fit the stationary ring with a “slip” or “tap” fit and the rotating ring with enough interference to prevent relative motion during operation. Recommended fits depend on bearing type, size, and tolerance grade. For example, a typical ABEC-1 ball-bearing fit would be in the range of 0.0005-in. clearance for the stationary ring and 0.0005-in. interference for the rotating ring. Manufacturing tolerances on the shaft and housing bearing interface dimensions are typically in the range of 0.0003 in. for ABEC-1 ball bearings. Proper fits and tolerances are influenced by the radial stiffness of the shaft and housing, and sometimes by thermal expansion.

It is important to recognize that bearing-fit pressures influence the internal fits between the balls or rollers and their races. Too tight a fit can cause internal interference that shortens the bearing life.

Care must be taken in the installation and removal of bearings to ensure that the required forces are applied directly to the bearing ring. If these forces are transmitted through the bearing, as when applying a force on the outer race to force the bearing into place on the shaft, the bearing can be damaged. Interference-fit installations are sometimes facilitated by heating the outer member or by “freezing” the inner member, by packing it in dry ice (solidified carbon dioxide). Any heating of the bearing must not be sufficient to damage the steel or any preinstalled lubricant.

Detailed information regarding bearing fits is contained in bearing manufacturers’ literature and in ANSI and AFBMA standards. Adhesives are sometimes used together with slightly reduced tolerances on the shaft and housing bore dimensions, particularly for non-precision applications.

Bearing alignment

Ball and roller bearings are precision components and are manufactured with very small internal clearances. They therefore provide very accurate positioning of the shaft. However, due to the small clearances, they do not perform well if they are misaligned. If there are doubts concerning accurate alignment of the housing into which the bearing will fit (or the amount of deflection of the shaft on which they are mounted), **SELF-ALIGNING BEARINGS** may be specified. In these bearings, self-aligning is achieved by allowing the plane of the balls or rollers to tilt relative to the spherical race in the outer housing. Double row ball bearing (which has greater radial load capacity than a single-row bearing) allowing the outer race to tilt within a spherical seating. An alternative construction has an outer race with an external spherical surface, enclosed within an additional casing which has an internal spherical surface.

Lubrication of bearings

Lubrication of rolling contact bearings is important because it:

- Prevents corrosion.
- Greatly reduces the effects of the sliding friction present in all bearings,
- particularly roller bearings.
- Carries heat away from heavily loaded bearings.

Lubrication may be achieved by

- Drip, splash or mist (oil only).
- Lubricators (oil or grease).

- Oil circulation within a gearbox by entrainment and splashing, etc.
- Grease is a suitable lubricant provided rotational speed is not too high and mineral oils are good general purpose bearing lubricants, although many modern applications demand **special high-pressure lubricants.**

Sealing:

Good sealing is essential. Seals should:

- Keep dirt out, thereby preventing premature wear of the bearing.
- Keep lubricant in, ensuring that the bearing (and possibly other components) will not run short of lubricant.

Bearings Design:

Bearing Life

When the ball or roller of rolling-contact bearings rolls, contact stresses occur on the inner ring, the rolling element, and on the outer ring. Because the curvature of the contacting elements in the axial direction is different from that in the radial direction, the equations for these stresses are more involved than in the Hertz equations. If a bearing is clean and properly lubricated, is mounted and sealed against the entrance of dust and dirt, is maintained in this condition, and is operated at reasonable temperatures, then metal fatigue will be the only cause of failure. Inasmuch as metal fatigue implies many millions of stress applications successfully endured, we need a quantitative life measure. Common life measures are

• Number of revolutions of the inner ring (outer ring stationary) until the first tangible evidence of fatigue

• Number of hours of use at a standard angular speed until the first tangible evidence of fatigue

The commonly used term is *bearing life,* which is applied to either of the measures just mentioned. It is important to realize, as in all fatigue, life as defined above is a stochastic variable and, as such, has both a distribution and associated statistical parameters. The life measure of an individual bearing is defined as the total number of revolutions (or hours at a constant speed) of bearing operation until the failure criterion is developed. Under ideal conditions, the fatigue failure consists of spalling of the load carrying surfaces. The American Bearing Manufacturers Association (ABMA) standard states that the failure criterion is the first evidence of fatigue. The fatigue criterion used by the Timken Company laboratories is the spalling or pitting of an area of 0.01 in^2. Timken also observes that the useful life of the bearing may extend considerably beyond this point. This is an operational definition of fatigue failure in rolling bearings.

The ***rating life*** is a term sanctioned by the **ABMA** and used by most manufacturers. The rating life of a group of nominally identical ball or roller bearings is defined as the number of revolutions (or hours at a constant speed) that 90 percent of a group of bearings will achieve or exceed before the failure criterion develops. The terms ***minimum life,*** ***L_{10} life,*** **and** ***B_{10} life*** are also used as synonyms for rating life. The rating life is the 10^{th} percentile location of the bearing group's revolutions-to-failure distribution.

Median life is the 50th percentile life of a group of bearings. The term ***average life*** has been used as a synonym for median life, contributing to confusion. When many groups of bearings are tested, the median life is between 4 and 5 times the L_{10} life. Each bearing manufacturer will choose a specific rating life for which load ratings of its bearings are reported. The most commonly used rating life is 10^6 revolutions. The Timken Company is a well-known exception, rating its bearings at 3 000 hours at 500 rev/min, which is $90(10^6)$ revolutions. These levels of rating life are actually quite low for today's bearings, but since rating life is an arbitrary reference point, the traditional values have generally been maintained.

Bearing Load Life at Rated Reliability

A *catalog load rating* is defined as the radial load that causes 10 percent of a group of bearings to fail at the bearing manufacturer's rating life. We shall denote the catalog load rating as C_{10} or C. The catalog load rating is often referred to as a *Basic Dynamic Load Rating,* or sometimes just Basic Load Rating, if the manufacturer's rating life is 10^6 revolutions.

The radial load that would be necessary to cause failure at such a low life would be unrealistically high. Consequently, the Basic Load Rating should be viewed as a reference value, and not as an actual load to be achieved by a bearing.

"Catalogue Information" for Rolling-Element Bearings

Bearing manufacturers' catalogues identify bearings by number, give complete dimensional information, list rated load capacities, and furnish details concerning mounting, lubrication, and operation. Dimensions of the more common series of radial ball bearings, angular ball bearings, and cylindrical roller bearings are given in the following table and illustrated in the figure. For bearings of these types having bores of 20 mm and larger, the bore diameter is five times the last two digits in the bearing number. For example, No. L08 is an extra-light series bearing with a 40-mm bore, No. 316 is a medium series with an 80-mm bore, and so on. Actual bearing numbers include additional letters and numbers to provide more information. Many bearing varieties are also available in inch series.

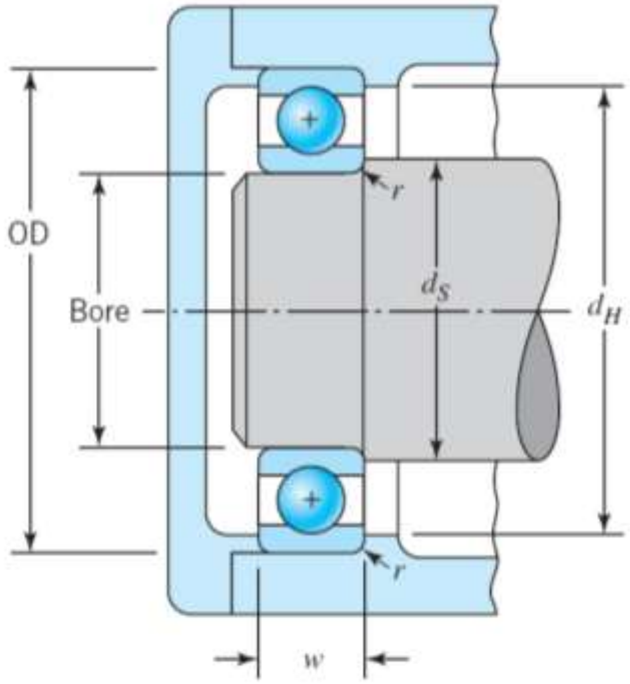


Figure: Shaft and housing shoulder dimensions

TABLE 14.1 Bearing Dimensions

Bearings Basic Number	Bore (mm)	Ball Bearings					Roller Bearing				
		OD (mm)	w (mm)	r^a (mm)	d_S (mm)	d_H (mm)	OD (mm)	w (mm)	r^a (mm)	d_S (mm)	d_H (mm)
L00	10	26	8	0.30	12.7	23.4					
200	10	30	9	0.64	13.8	26.7					
300	10	35	11	0.64	14.8	31.2					
L01	12	28	8	0.30	14.5	25.4					
201	12	32	10	0.64	16.2	28.4					
301	12	37	12	1.02	17.7	32.0					
L02	15	32	9	0.30	17.5	29.2					
202	15	35	11	0.64	19.0	31.2					
302	15	42	13	1.02	21.2	36.6					
L03	17	35	10	0.30	19.8	32.3	35	10	0.64	20.8	32.0
203	17	40	12	0.64	22.4	34.8	40	12	0.64	20.8	36.3
303	17	47	14	1.02	23.6	41.1	47	14	1.02	22.9	41.4
L04	20	42	12	0.64	23.9	38.1	42	12	0.64	24.4	36.8
204	20	47	14	1.02	25.9	41.7	47	14	1.02	25.9	42.7
304	20	52	15	1.02	27.7	45.2	52	15	1.02	25.9	46.2
L05	25	47	12	0.64	29.0	42.9	47	12	0.64	29.2	43.4
205	25	52	15	1.02	30.5	46.7	52	15	1.02	30.5	47.0
305	25	62	17	1.02	33.0	54.9	62	17	1.02	31.5	55.9
L06	30	55	13	1.02	34.8	49.3	47	9	0.38	33.3	43.9
206	30	62	16	1.02	36.8	55.4	62	16	1.02	36.1	56.4
306	30	72	19	1.02	38.4	64.8	72	19	1.52	37.8	64.0
L07	35	62	14	1.02	40.1	56.1	55	10	0.64	39.4	50.8
207.	35	72	17	1.02	65.0	42.4	72	17	1.02	41.7	65.3
307	35	80	21	1.52	45.2	70.4	80	21	1.52	43.7	71.4
L08	40	68	15	1.02	45.2	62.0	68	15	1.02	45.7	62.7
208	40	80	18	1.02	48.0	72.4	80	18	1.52	47.2	72.9

Bearings Basic Number	Bore (mm)	Ball Bearings					Roller Bearing				
		OD (mm)	w (mm)	r^a (mm)	d_S (mm)	d_H (mm)	OD (mm)	w (mm)	r^a (mm)	d_S (mm)	d_H (mm)
308	40	90	23	1.52	50.8	80.0	90	23	1.52	49.0	81.3
L09	45	75	16	1.02	50.8	68.6	75	16	1.02	50.8	69.3
209	45	85	19	1.02	52.8	77.5	85	19	1.52	52.8	78.2
309	45	100	25	1.52	57.2	88.9	100	25	2.03	55.9	90.4
L10	50	80	16	1.02	55.6	73.7	72	12	0.64	54.1	68.1
210	50	90	20	1.02	57.7	82.3	90	20	1.52	57.7	82.8
310	50	110	27	2.03	64.3	96.5	110	27	2.03	61.0	99.1
L11	55	90	18	1.02	61.7	83.1	90	18	1.52	62.0	83.6
211	55	100	21	1.52	65.0	90.2	100	21	2.03	64.0	91.4
311	55	120	29	2.03	69.8	106.2	120	29	2.03	66.5	108.7
L12	60	95	18	1.02	66.8	87.9	95	18	1.52	67.1	88.6
212	60	110	22	1.52	70.6	99.3	110	22	2.03	69.3	101.3
312	60	130	31	2.03	75.4	115.6	130	31	2.54	72.9	117.9
L13	65	100	18	1.02	71.9	92.7	100	18	1.52	72.1	93.7
213	65	120	23	1.52	76.5	108.7	120	23	2.54	77.0	110.0
313	65	140	33	2.03	81.3	125.0	140	33	2.54	78.7	127.0
L14	70	110	20	1.02	77.7	102.1	110	20	Not Available		
214	70	125	24	1.52	81.0	114.0	125	24	2.54	81.8	115.6
314	70	150	35	2.03	86.9	134.4	150	35	3.18	84.3	135.6
L15	75	115	20	1.02	82.3	107.2	115	20	Not Available		
215	75	130	25	1.52	86.1	118.9	130	25	2.54	85.6	120.1
315	75	160	37	2.03	92.7	143.8	160	37	3.18	90.4	145.8
L16	80	125	22	1.02	88.1	116.3	125	22	2.03	88.4	117.6
216	80	140	26	2.03	93.2	126.7	140	26	2.54	91.2	129.3
316	80	170	39	2.03	98.6	152.9	170	39	3.18	96.0	154.4
L17	85	130	22	1.02	93.2	121.4	130	22	2.03	93.5	122.7
217	85	150	28	2.03	99.1	135.6	150	28	3.18	98.0	139.2
317	85	180	41	2.54	105.7	160.8	180	41	3.96	102.9	164.3
L18	90	140	24	1.52	99.6	129.0	140	24	Not Available		
218	90	160	30	2.03	104.4	145.5	160	30	3.18	103.1	147.6
318	90	190	43	2.54	111.3	170.2	190	43	3.96	108.2	172.7
L19	95	145	24	1.52	104.4	134.1	145	24	Not Available		
219	95	170	32	2.03	110.2	154.9	170	32	3.18	109.0	157.0
319	95	200	45	2.54	117.3	179.3	200	45	3.96	115.1	181.9
L20	100	150	24	1.52	109.5	139.2	150	24	2.54	109.5	141.7
220	100	180	34	2.03	116.1	164.1	180	34	3.96	116.1	167.1
320	100	215	47	2.54	122.9	194.1	215	47	4.75	122.4	194.6
L21	105	160	26	2.03	116.1	146.8	160	26	Not Available		
221	105	190	36	2.03	121.9	173.5	190	36	3.96	121.4	175.3
321	105	225	49	2.54	128.8	203.5	225	49	4.75	128.0	203.5
L22	110	170	28	2.03	122.7	156.5	170	28	2.54	121.9	159.3
222	110	200	38	2.03	127.8	182.6	200	38	3.96	127.3	183.9
322	110	240	50	2.54	134.4	218.2	240	50	4.75	135.9	217.2
L24	120	180	28	2.03	132.6	166.6	180	28	Not Available		
224	120	215	40	2.03	138.2	197.1	215	40	4.75	139.2	198.9
324	120	Not Available					260	55	6.35	147.8	235.2
L26	130	200	33	2.03	143.8	185.4	200	33	3.18	143.0	188.2
226	130	230	40	2.54	149.9	210.1	230	40	4.75	149.1	213.9
326	130	280	58	3.05	160.0	253.0	280	58	6.35	160.3	254.5
L28	140	210	33	2.03	153.7	195.3	210	33	Not Available		
228	140	250	42	2.54	161.5	228.6	250	42	4.75	161.5	232.4
328	140	Not Available					300	62	7.92	172.0	271.3
L30	150	225	35	2.03	164.3	209.8	225	35	3.96	164.3	212.3

Bearings Basic Number	Bore (mm)	**Ball Bearings**					**Roller Bearing**				
		OD (mm)	***w*** (mm)	***r*[a]** (mm)	**d_S** (mm)	**d_H** (mm)	**OD** (mm)	***w*** (mm)	***r*[a]** (mm)	**d_S** (mm)	**d_H** (mm)
230	150	270	45	2.54	173.0	247.6	270	45	6.35	174.2	251.0
L32	160	240	38	2.03	175.8	223.0	240	38	Not Available		
232	160	Not Available					290	48	6.35	185.7	269.5
L36	180	280	46	2.03	196.8	261.6	280	46	4.75	199.6	262.9
236	180	Not Available					320	52	6.35	207.5	298.2
L40	200						310	51	Not Available		
240	200	Not Available					360	58	7.92	232.4	334.5
L44	220						340	56	Not Available		
244	220	Not Available					400	65	9.52	256.0	372.1
L48	240						360	56	Not Available		
248	240	Not Available					440	72	9.52	279.4	408.4

[a] Maximum fillet radius on a shaft and in housing that will clear the bearing corner radius

The following Table lists *rated load capacities*, *C*. These values correspond to a constant radial load that 90 percent of a group of presumably identical bearings can endure for $9 * 10^7$ revolutions (as 3000 hours of 500-rpm operation) without the onset of surface fatigue failures.

Caution: Rated capacities given by different bearing manufacturers are not always directly comparable. The basis for ratings must always be checked.

Bearing Selection

For specific bearing application, we select the bearing type, grade of precision (usually ABEC 1), lubricant, closure (i.e., open, shielded, or sealed), and basic load rating. Often, special circumstances must be taken into account. For example, if the bearing will be subjected to a heavy load when not rotating, its *static load capacity* (given in bearing manufacturers' catalogues) should not be exceeded. Otherwise, the balls or rollers will slightly indent the rings. This is called brinelling because the indentations resemble marks produced by a Brinell hardness tester. The indentations will make subsequent rotation noisy. (If noise is not objectionable, the static capacity can often be exceeded by a factor of up to 3.) It is interesting that similar extremely slight indentation during rotation is not harmful because it leaves the ring surfaces smooth and annular. Another special consideration is maximum speed. The limitation is one of linear surface speed rather than rotating speed; hence, small bearings can operate at higher rpm than large bearings. Lubrication is especially important in high-speed bearing applications, the best being a fine oil mist or spray. This provides the necessary lubricant film and carries away friction heat with a minimum "churning loss" within the lubricant itself. For ball bearings, nonmetallic separators permit highest speeds. ABEC 1 precision single-row ball bearings with nonmetallic separators and oil mist lubrication can run at inner ring surface speeds up to 75 m/s and have a life of 3000 hours while carrying one-third of the rated load capacity This translates to a *DN value* (bore diameter in millimeters times rpm) of about $1.25 * 10^6$. For oil drip or splash lubrication this figure is reduced by about one-third, and for grease lubrication by about two-thirds. Under the most favorable conditions, roller bearings can operate up to a *DN* value of about 450,000. For applications with

extreme rotating speeds, it is advisable to consult the bearing manufacturer. In selecting bearings, attention should be given to possible misalignment and to sealing and lubrication. If temperatures are extreme, the bearing manufacturer should be consulted.
The size of bearing selected for an application is usually influenced by the size of shaft required (for strength and rigidity considerations) and by the available space. In addition, the bearing must have a high enough load rating to provide an acceptable combination of life and reliability. The major factors influencing the load rating requirement are discussed below.

Life Requirement

Bearing applications usually require lives different from that used for the catalogue rating. Palmgren determined that ball-bearing life varies inversely with approximately the third power of the load. Later studies have indicated that this exponent ranges between 3 and 4 for various rolling-element bearings. Many manufacturers retain Palmgren's exponent of 3 for ball bearings and use (10/3) for roller bearings.
Following the recommendation of other manufacturers, this note will use the exponent (3.33) for both bearing types. Thus

$$L = L_R(C/F_r)^{3.33}$$

or

$$C_{req} = F_r(L/L_R)^{0.3}$$

where

C = rated capacity (as from Table) and C_{req} = the required value of C for the application, L_R = life corresponding to rated capacity (i.e., 9 x 10^7 revolutions)
F_r = radial load involved in the application, L = life corresponding to radial load F_r , or life required by the application.
Thus doubling the load on a bearing reduces its life by a factor of about 10. Different manufacturers' catalogues use different values of L_R. Some use $L_R = 10^6$ revolutions. A quick calculation shows that the values in the Table must be multiplied by 3.86 to be comparable with ratings based on a life of 10^6 revolutions.

Reliability Requirement

Tests show that the *median* life of rolling-element bearings (ball bearings in particular) is about five times the standard 10 percent failure fatigue life. The standard life is commonly designated as the L_{10} *life* (sometimes as the B_{10} life). Since this life corresponds to 10 percent failures, it also means that this is the life for which 90 percent have *not* failed, and corresponds to *90 percent reliability. Thus, the life for 50 percent reliability is about five times the life for 90 percent reliability.*
Many designs require greater than 90 percent reliability. The distribution of fatigue lives of a group of presumably identical parts does not correspond to the normal distribution curve. Rather, fatigue lives characteristically have a skewed distribution. This corresponds generally with a mathematical formula proposed by W. Weibull of Sweden, known as the *Weibull distribution.* Using the general Weibull equation together with extensive experimental data, the AFBMA has formulated recommended *life adjustment reliability factors, Kr* , plotted in the following figure. These factors are applicable to both ball and roller bearings. The rated bearing life for any given

reliability (greater than 90 percent) is thus the product, $K_r L_R$. Incorporating this factor into the proceeding equations gives

$$L = K_r\, L_R\, (C/F_r)^{3.33}$$

or

$$C_{req} = F_r\, (L/K_r L_R)^{0.3}$$

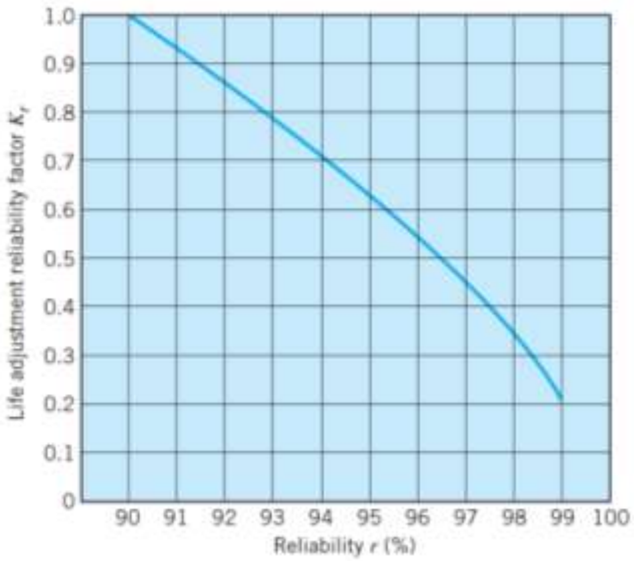


Figure: Reliability factor K_r

Influence of Axial Loading

Cylindrical roller bearings are very limited in their thrust capacity because axial loads produce sliding friction at the roller ends. Even so, when these bearings are properly aligned, radially loaded, and oil-lubricated, they can carry thrust loads up to 20 percent of their rated radial capacities. This enables pairs of cylindrical roller bearings to support shafts subjected to light thrust, as by spur gears or chain sprockets. Tapered roller bearings can, of course, carry substantial thrust as well as radial loads. For ball bearings, any combination of radial load (F_r) and thrust load (F_t) results in approximately the same life as does a pure radial ***equivalent load***, F_e, calculated from the equations that follow. Load angle (α) is defined before. Radial bearings have a zero load angle. Standard values of (α) for angular ball bearings are 15°, 25°, and 35°. Space permits including only the treatment of 25° angular ball bearings in this notes.

For $\alpha=0°$ **(radial ball bearings)**

For $0 < F_t/F_r < 0.35$, $F_e = F_r$

For $0.35 < F_t/F_r < 10$ $F_e = F_r\,[1 + 1.115\,(F_t/F_r - 0.35)]$

For $F_t/F_r > 10$, $F_e = 1.176\, F_t$

For $\alpha=25°$ **(angular ball bearings)**

For $0 < F_t/F_r < 0.68$, $F_e = Fr$

For $0.68 < F_t/F_r < 10$ $Fe = F_r\,[1 + 0{,}870\,(F_t/F_r - 0.68)]$

For $F_t/F_r > 10$, $F_e = 0.911\, F_t$

Influence of elevated Temperature

At elevated temperature dynamics load carrying capacity is reduced. The reduction in the capacity at different temperatures is taken into account by multiplying the rated capacity C by a temperature factor (K_t) obtained from the following table:

Table: Temperature factor (K_t)

Bearing temperature [°C]	150	200	250	300
Temperature factor (K_t)	1.00	0.90	0.75	0.60

Influence of Shock Loading

The standard bearing rated capacity is for the condition of uniform load without shock. This desirable condition may prevail for some applications (such as bearings on the motor and rotor shafts of a belt-driven electric blower), but other applications have various degrees of shock loading. This has the effect of increasing the nominal load by an ***application factor K_a***. Experience within the specific industry is the best guide. The Table gives representative sample values.

The load-application factors in next Table serve the same purpose as factors of safety; use them to increase the equivalent load before selecting a bearing.

Table: Application Factors K_a

Type of Application	Ball Bearing	Roller Bearing
Uniform load, no impact	1.0	1.0
Gearing	1.0–1.3	1.0
Light impact	1.2–1.5	1.0–1.1
Moderate impact	1.5–2.0	1.1–1.5
Heavy impact	2.0–3.0	1.5–2.0

Substituting F_e for F_r and adding K_a modifies Equations to give

$$L = K_r K_t L_R (C/F_e K_a)^{3.33}$$

or

$$C_{req} = F_e K_t K_a (L/K_r L_R)^{0.3}$$

When the preceding equations are used, the question is what life, L should be required. The Table of typical bearing life (next section) may be used as a guide when more specific information is not available. It is worth noting that the useful life of a bearing in industrial applications where noise is not a factor may extend significantly beyond the appearance of the first small area of surface fatigue damage, which is the failure criterion in standard tests.

Typical Bearing Life for Various Design Applications

To assist the designer in the selection of bearings, most of the manufacturers' handbooks contain data on bearing life for many classes of machinery, as well as information on load-application factors. Such information has been accumulated the hard way, that is, by experience, and the beginner designer should utilize this information until he or she gains enough experience to know when deviations are possible.

The following Table contains recommendations on bearing life for some classes of machinery. When the preceding equations are used, the question is what life, L should be required. The following Table may be used as a guide when more specific information is not available. (It is worth noting that the useful life of a bearing in industrial applications where noise is not a factor may extend significantly beyond the appearance of the first small area of surface fatigue damage, which is the failure criterion in standard tests.)

Bearing manufacturers formerly reduced life ratings when the outer ring rotated relative to the load (as with a trailer wheel, rotating around a fixed spindle). As a result of more recent evidence, this is no longer done. If both rings rotate, the relative rotation between the two is used in making life calculations.

Many applications involve loads that vary with time. In such cases, the Palmgren linear cumulative-damage rule is applicable.

TABLE: Bearing Rated Capacities, ***C***, for L_R 90 x 10^6 Revolution Life with 90 Percent Reliability

Bore	Radial Ball,			Angular Ball,			Roller		
	L00	200	300	L00	200	300	1000	1200	1300
	Xlt	lt	med	Xlt	lt	med	Xlt	lt	med
(mm)	(kN)	(kN)	(kN)	(kN)	(kN)	(kN)	(kN)	(kN)	(kN)
10	1.02	1.42	1.90	1.02	1.10	1.88			
12	1.12	1.42	2.46	1.10	1.54	2.05			
15	1.22	1.56	3.05	1.28	1.66	2.85			
17	1.32	2.70	3.75	1.36	2.20	3.55	2.12	3.80	4.90
20	2.25	3.35	5.30	2.20	3.05	5.80	3.30	4.40	6.20
25	2.45	3.65	5.90	2.65	3.25	7.20	3.70	5.50	8.50
30	3.35	5.40	8.80	3.60	6.00	8.80	2.40[a]	8.30	10.0
35	4.20	8.50	10.6	4.75	8.20	11.0	3.10[a]	9.30	13.1
40	4.50	9.40	12.6	4.95	9.90	13.2	7.20	11.1	16.5
45	5.80	9.10	14.8	6.30	10.4	16.4	7.40	12.2	20.9
50	6.10	9.70	15.8	6.60	11.0	19.2	5.10[a]	12.5	24.5
55	8.20	12.0	18.0	9.00	13.6	21.5	11.3	14.9	27.1
60	8.70	13.6	20.0	9.70	16.4	24.0	12.0	18.9	32.5
65	9.10	16.0	22.0	10.2	19.2	26.5	12.2	21.1	38.3
70	11.6	17.0	24.5	13.4	19.2	29.5		23.6	44.0
75	12.2	17.0	25.5	13.8	20.0	32.5		23.6	45.4
80	14.2	18.4	28.0	16.6	22.5	35.5	17.3	26.2	51.6
85	15.0	22.5	30.0	17.2	26.5	38.5	18.0	30.7	55.2
90	17.2	25.0	32.5	20.0	28.0	41.5		37.4	65.8
95	18.0	27.5	38.0	21.0	31.0	45.5		44.0	65.8
100	18.0	30.5	40.5	21.5	34.5	20.9		48.0	72.9
105	21.0	32.0	43.5	24.5	37.5			49.8	84.5
110	23.5	35.0	46.0	27.5	41.0	55.0	29.4	54.3	85.4
120	24.5	37.5		28.5	44.5			61.4	100.1
130	29.5	41.0		33.5	48.0	71.0	48.9	69.4	120.1
140	30.5	47.5		35.0	56.0			77.4	131.2
150	34.5			39.0	62.0		58.7	83.6	
160	113.4								
180	47.0			54.0			97.9	140.1	
200								162.4	
220								211.3	
240								258.0	

[a] 1000 (Xlt) series bearings are not available in these sizes. Capacities shown are for the 1900 (XXlt) series. *Source:* New Departure–Hyatt Bearing Division, General Motors Corporation.

Table: Typical Bearing Life for Various Design Applications

Uses	Design life [in hours]	Uses	Design life [in hours]
Agricultural equipment	3000 - 6000	**Gearing units**	
Aircraft equipment	500 - 2000	Automotive	600 - 5000
Automotive		Multipurpose	8000 - 15000
Race car	500 - 800	Machine tools	20000
Light motor cycle	600 - 1200	Rail Vehicles	15000 - 25000
Heavy motor cycle	1000 - 2000	Heavy rolling mill	> 50000
Light cars	1000 - 2000	**Machines**	
Heavy cars	1500 - 2500	Beater mills	20000 - 30000
Light trucks	1500 - 2500	Briquette presses	20000 - 30000
Heavy trucks	2000 - 2500	Grinding spindles	1000 - 2000
Buses	2000 - 5000	Machine tools	10000 - 30000
Electrical		Mining machinery	4000 - 15000
Household appliances	1000 - 2000	Paper machines	50000 - 80000
Motors ≤ 1/2 hp	1000 - 2000	**Rolling mills**	
Motors ≤ 3 hp	8000 - 10000	Small cold mills	5000 - 6000
Motors, medium	10000 - 15000	Large multipurpose mills	8000 - 10000
Motors, large	20000 - 30000	**Rail vehicle axle**	
Elevator cables sheaves	40000 - 60000	Mining cars	5000
Mine ventilation fans	40000 - 50000	Motor rail cars	16000 - 20000
Propeller thrust bearings	15000 - 25000	Open-pit mining cars	20000 - 25000
Propeller shaft bearings	> 80000	Streetcars	20000 - 25000
Gear drives		Passenger cars	26000
Boat gearing units	3000 - 5000	Freight cars	35000
Gear drives	> 50000	Locomotive outer bearings	20000 - 25000
Ship gear drives	20000 - 30000	Locomotive inner bearings	30000 - 40000
Machinery for 8 hour service which are not always fully utilized	14000 - 20000	Machinery for short or intermittent operation where service interruption is of minor importance	4000 - 8000
Machinery for 8 hour service which are fully utilized	20000 - 30000	Machinery for intermittent service where reliable operation is of great importance	8000 - 14000
Machinery for continuous 24 hour service	50000 - 60000	Instruments and apparatus in frequent use	0 - 500

Design procedure for bearing:

1. Find the equivalent load $\mathbf{F_e}$ that considering both radial and thrust load.

For $\alpha=0°$ **(radial ball bearings)**

For $0 < \mathbf{F_t/F_r} < 0.35$, $\mathbf{F_e} = \mathbf{F_r}$

For $0.35 < \mathbf{F_t/F_r} < 10$ $\mathbf{F_e} = \mathbf{F_r}\,[1 + 1.115\,(\mathbf{F_t/F_r} - 0.35)]$

For $\mathbf{F_t/F_r} > 10$, $\mathbf{F_e} = 1.176\,\mathbf{F_t}$

For $\alpha=25°$ **(angular ball bearings)**

For $0 < \mathbf{F_t/F_r} < 0.68$, $\mathbf{F_e} = \mathbf{Fr}$

For $0.68 < \mathbf{F_t/F_r} < 10$ **Fe** = F_r [1 + 0,870 ($\mathbf{F_t/F_r}$- 0.68)]

For $\mathbf{F_t/F_r} > 10$, $\mathbf{F_e} = 0.911\ \mathbf{F_t}$

2. Find the Application Factors K_a from table according to the loading conditions.

3. Find the Temperature Factors K_t from table according to the bearing temperature.

4.. Estimate the required life that matches with the application in according to the table of typical bearing life.

5. Find ***Reliability factor*** K_r from the figure according to the reliability requirements.

6. Use L_R as required.

7. Find the required capacity C_{req} from the equation:

$$C_{req} = F_e K_t K_a (L/K_r L_R)^{0.3}$$

8. Select the suitable bearing from the manufacture's catalogue or the table of the Bearing Rated Capacities, ***C.***

9. Extract all dimensions of the select bearing from the table of bearing dimensions.

Couplings

Couplings: They are used to connect sections of shafts or connect the shaft of a driving machine to the shaft of a driven machine. This afford a permanent connection, contrasted with clutches which provide for engagement or disengagement at will. Couplings can be either rigid or flexible. Rigid coupling are used for accurately aligned shafts. Flexible couplings are used to take care of small amount of misalignment, to provide axial movement of a shaft or to absorb some of the vibration of the system. Shafts axes with couplings can be collinear(e.g. Flanged coupling), intersect(e.g. Universal coupling) or parallel but not collinear(e.g. Oldham coupling). Couplings are usually designed according to torsion only.

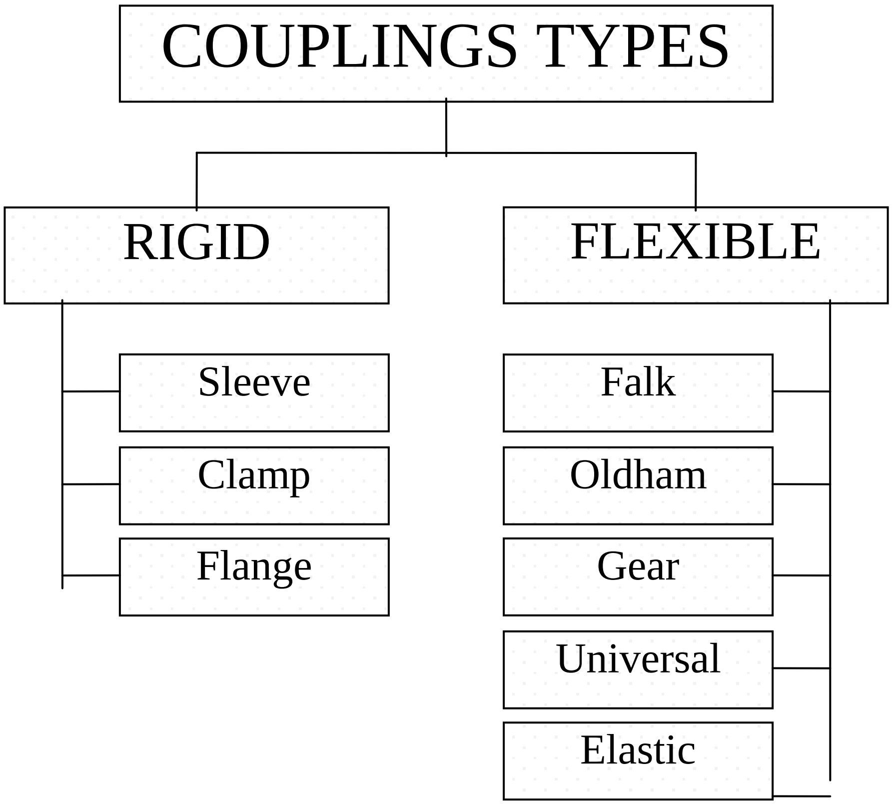
Couplings connect coaxial shafts. They are formed by two discs attached to shafts through key and jointed by bolts, parallel to shaft axis. The discs are made as flanges integral with the hub. The flanges are often made in cast iron. Muff couplings are thick cylinders which could be used as sleeves or split to be bolted around the shaft. The driving force in muff coupling is friction between the inner surface of muff and outer surface of shaft. The muff can be a single piece sleeve keyed to shafts or split in halves which are tightened by the bolts. The muff is made in cast iron.

Couplings

• Couplings transmit torque and motion between shafts in the presence of various types of misalignment

• Types of Misalignment

– Angular
– Parallel
– Torsional
– Axial

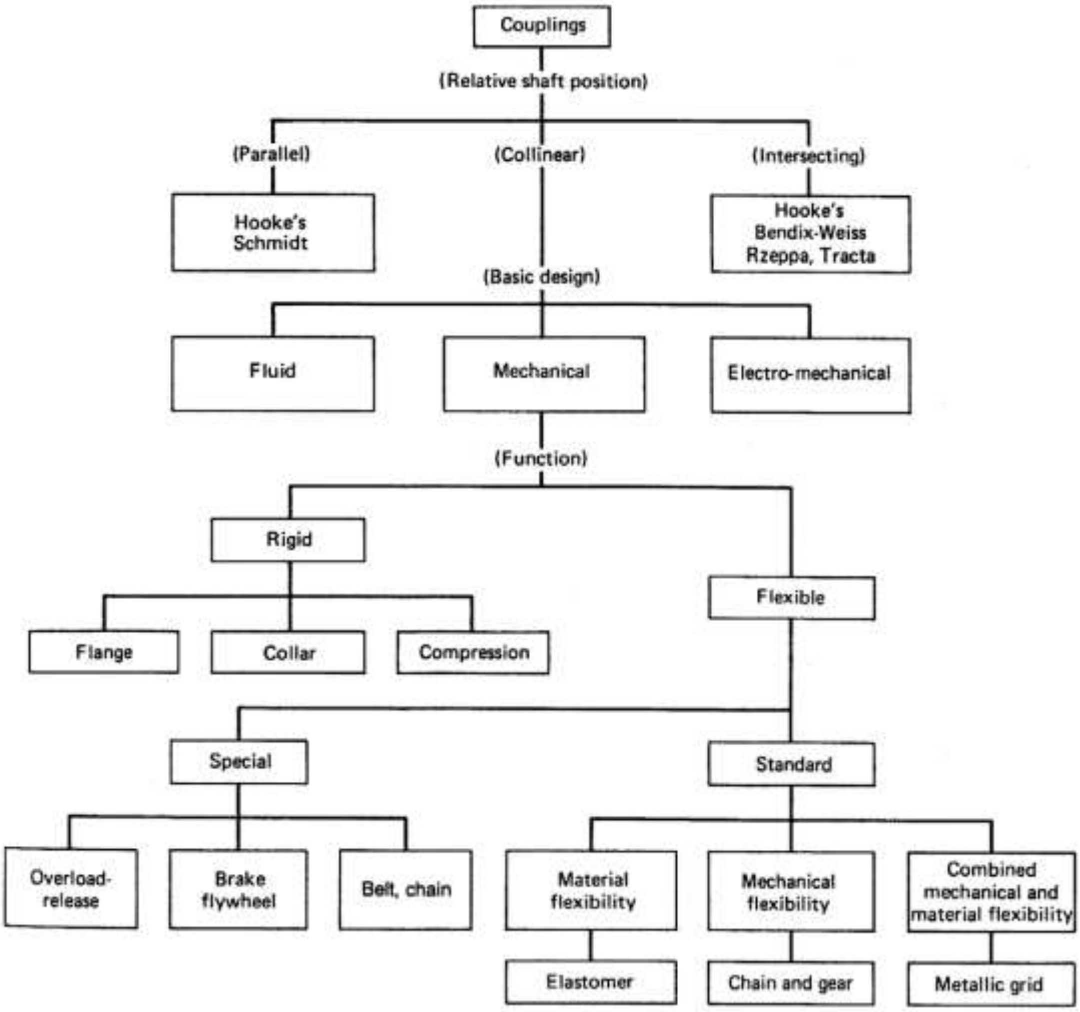


Types of Couplings

- Rigid Couplings
  - Set-screw
  - Keyed
  - Clamped
- Flexible Couplings
  - Jaw type
  - Gear, spline, grid, chain
  - Helical and bellows
  - Linkages
  - Universal Joints
- Used in pairs
- Basic Specs Include: nominal and peak torque, misalignment tolerances, shaft size, operating temp, speed range, and backlash.

Table 9-7 Characteristics of Various Types of Couplings

	Misalignment Tolerated				
Class	Axial	Angular	Parallel	Torsional	Comments
Rigid	large	none	none	none	requires accurate alignment
Jaw	slight	slight (<2°)	slight (3% *d*)	moderate	shock absorption—significant backlash
Gear	large	slight (<5°)	slight (<1/2% *d*)	none	slight backlash—large torque capacity
Spline	large	none	none	none	slight backlash—large torque capacity
Helical	slight	large (20°)	slight (<1% *d*)	none	one piece - compact—no backlash
Bellows	slight	large (17°)	moderate (20% *d*)	none	subject to fatigue failure
Flexible disc	slight	slight (3°)	slight (2% *d*)	slight to none	shock absorption—no backlash
Linkage (Schmidt)	none	slight (5°)	large (200% *d*)	none	no backlash—no sideloads on shaft
Hooke	none	large	large (in pairs)	none	slight backlash—speed variation unless used in pairs
Rzeppa	none	large	none	none	constant velocity


Couplings
(Relative shaft position)
(Parallel)
(Collinear)
(Intersecting)
Hooke's
Schmidt
Hooke's
Bendix-Weiss
Rzeppa, Tracta
(Basic design)
Fluid
Mechanical
Electro-mechanical
(Function)
Rigid
Flexible
Flange
Collar
Compression
Special
Standard
Overload-release
Brake flywheel
Belt, chain
Material flexibility
Mechanical flexibility
Combined mechanical and material flexibility
Elastomer
Chain and gear
Metallic grid

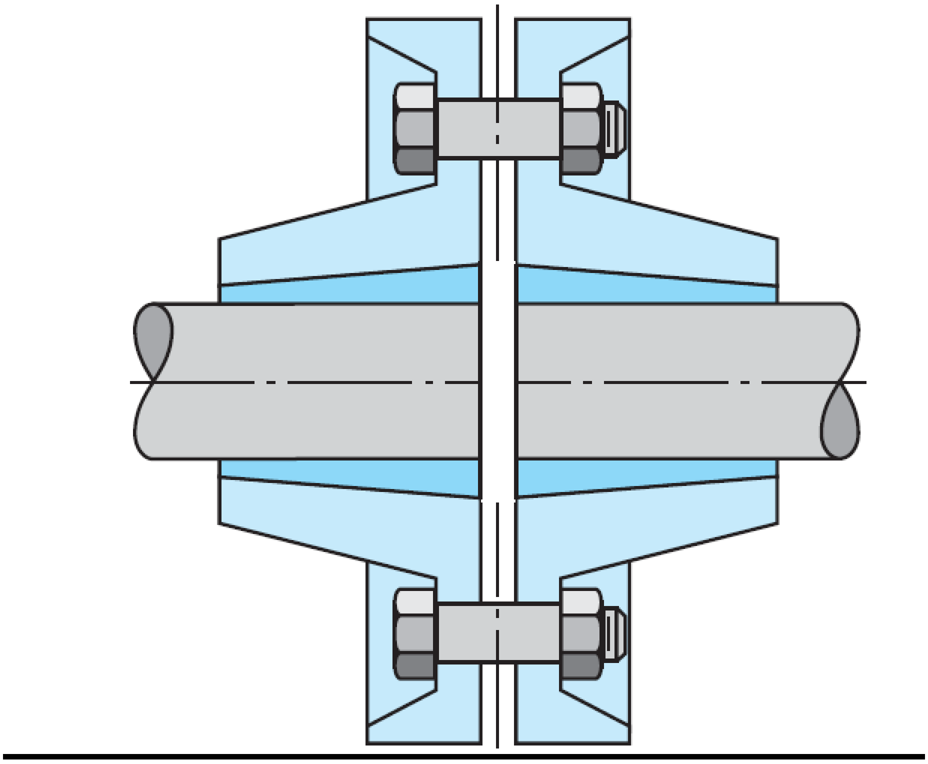


FIGURE :Rigid shaft coupling

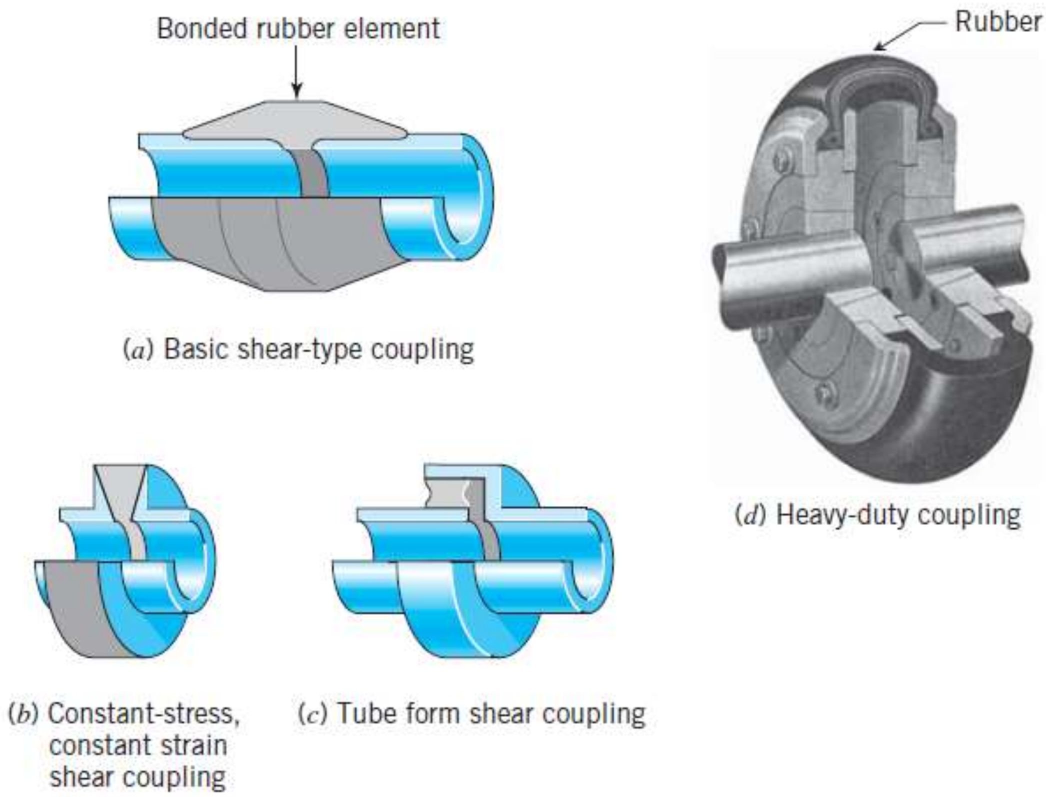


FIGURE : Rubber element flexible couplings. (*a*, *b*, *c*, Courtesy Lord Corporation. *d*, Courtesy Reliance Electric Company.)

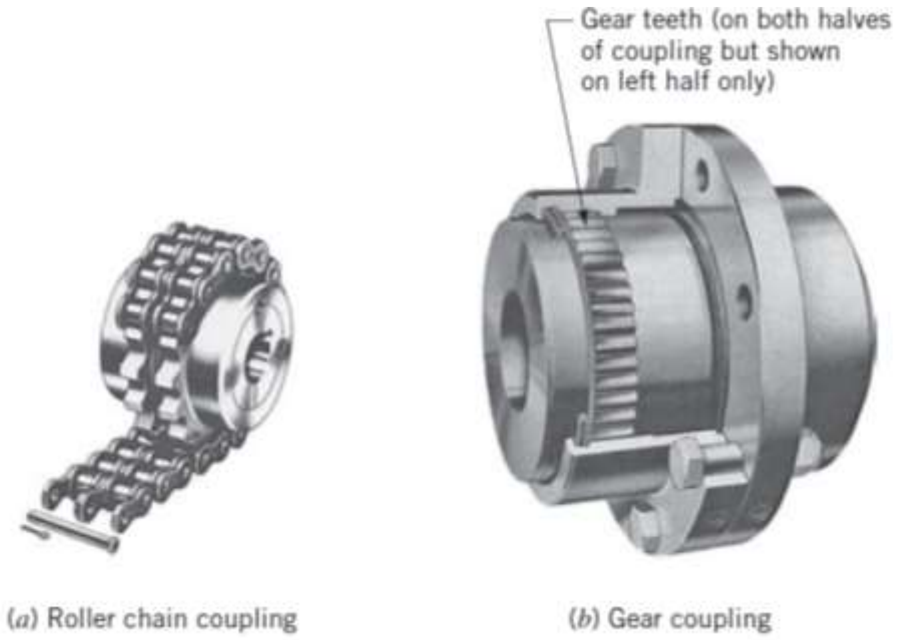


FIGURE : Metallic element flexible couplings. (Courtesy Reliance Electric Company.)

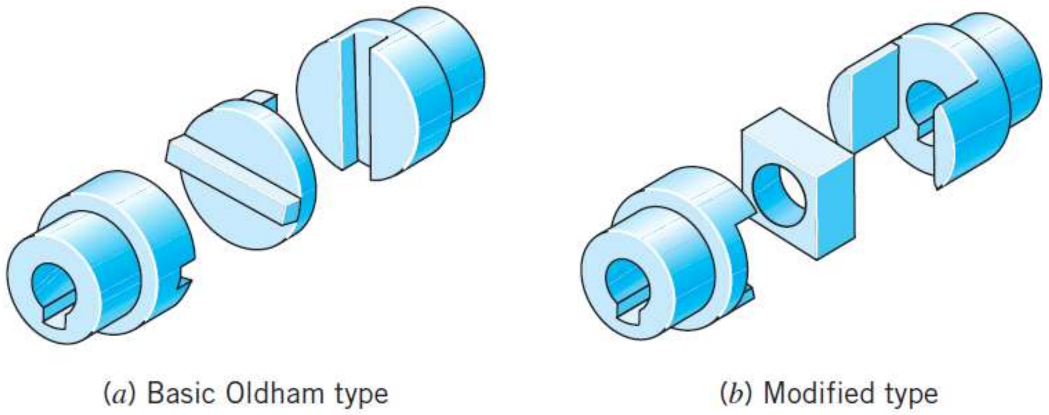


FIGURE : Oldham or slider block couplings. Both versions have a freely sliding center slider block that provides pairs of sliding surfaces at 90° orientation. The greater the shaft misalignment, the greater the sliding. Lubrication and wear must be considered.

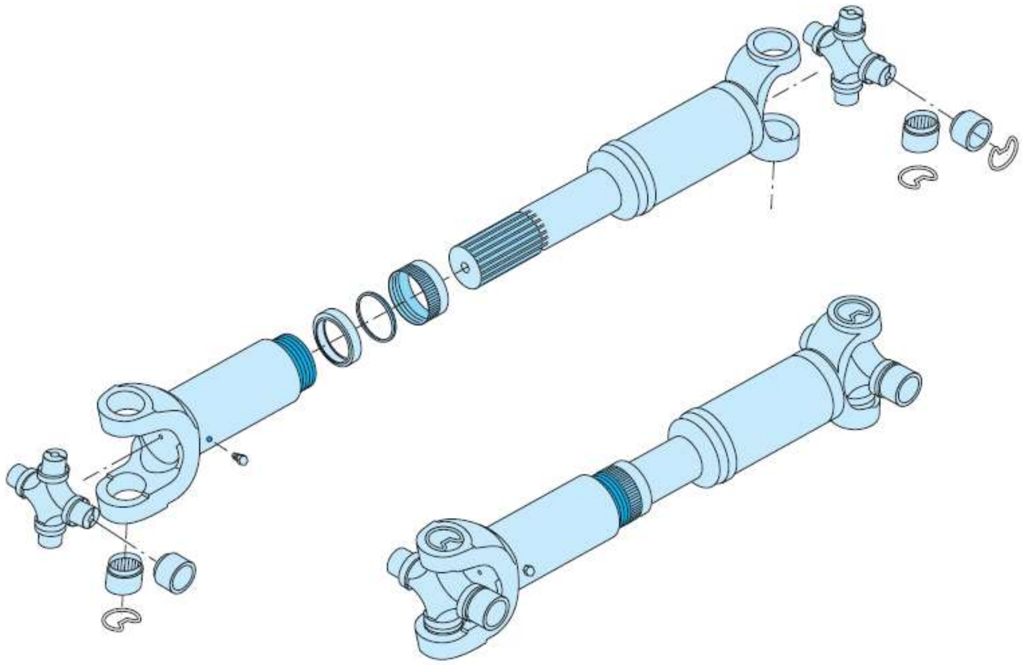


Figure: Cross-type universal joints. (Courtesy Dana Corporation.)

DESIGN OF SPRINGS

Springs are flexible machine elements used to exert force and store energy. A spring is an elastic object used to store mechanical energy. Springs are elastic bodies (generally metal) that can be twisted, pulled, or stretched by some force. They can return to their original shape when the force is released. In other words it is also termed as a resilient member. A spring is a flexible element used to exert a force or a torque and, at the same time, to store energy.

The force can be a linear push or pull, or it can be radial, acting similarly to a rubber band around a roll of drawings. The torque can be used to cause a rotation, for example, to close a door on a cabinet or to provide a counterbalance force for a machine element pivoting on a hinge.

Objectives of spring

- **To provide Cushioning, to absorb, or to control the energy due to shock and vibration: Car springs or railway buffers to control energy, springs-supports and vibration dampers.**
- **To Control motion: Maintaining contact between two elements (cam and its follower). Creation of the necessary pressure in a friction device (a brake or a clutch)**
- **To Measure forces: Spring balances, gages**

Springs classification: Springs can be classified according to the direction and the nature of the force exerted by the spring when it is deflected (see the following table).

Table: Springs classification

Uses	Types of springs
Push	Helical compression spring, Belleville spring, Torsion spring, force acting at the end of torque arm. flat spring, such as a cantilever spring or leaf spring
Pull	Helical extension spring, Torsion spring, force acting at the end of torque arm. Flat spring, such as a cantilever spring or leaf spring, Draw bar spring (special case of the compression spring) constant – force spring.
Radial torque	Garter spring, elastomeric band, spring clamp, Torsion spring, Power spring

In general, springs may be classified as wire springs, flat springs, or special shaped springs, and there are variations within these divisions. Wire springs include helical springs of round or square wire, made to resist and deflect under tensile, compressive, or torsional loads. Flat springs include cantilever and elliptical types, wound motor- or clock-type power springs, and flat spring washers, usually called Belleville springs.

Spring manufacturing processes:

If springs are of very small diameter and the wire diameter is also small then the springs are normally manufactured by a cold drawn process through a mangle. However, for very large springs having also large coil diameter and wire diameter one has to go for manufacture by hot processes. First one has to heat the wire and then use a proper mangle to wind the coils. Two types of springs which are mainly used are, helical springs and leaf springs. We shall consider in this course the design aspects of two types of springs.

HELICAL SPRING:

Definition: It is made of wire coiled in the form of helix having circular, square or rectangular cross section.

Terminology of helical spring:

The main dimensions of a helical spring subjected to compressive force are shown in the figure. They are as follows:

d = wire diameter of spring (mm)
D_i = inside diameter of spring coil (mm)
D_o =outside diameter of spring coil (mm)
D = mean coil diameter (mm)

Therefore $D = (D_i + D_o)/2$

There is an important parameter in spring design called **spring index**. It is denoted by letter C. The spring index is defined as the ratio of mean coil diameter to wire diameter. Or $C = D/d$

In design of helical springs, the designer should use good judgment in assuming the value of the spring index C. The spring index indicates the relative sharpness of the curvature of the coil.

A low spring index means high sharpness of curvature. When the spring index is low ($C < 3$), the actual stresses in the wire are excessive due to curvature effect. Such a spring is difficult to manufacture and special care in coiling is required to avoid cracking in some wires. When the spring index is high ($C > 15$), it results in large variation in coil diameter. Such a spring is prone to buckling and also tangles easily during handling. Spring index from 4 to 12 is considered better from manufacturing considerations. Therefore, in practical applications, the spring index in the range of 6 to 9 is still preferred particularly for close tolerance springs and those subjected to cyclic loading.

There are three terms - free length, compressed length and solid length that are illustrated in the figure. These terms are related to helical compression spring. These lengths are determined by following way

1) **Solid length:** solid length is defined as the axial length of the spring which is so compressed, that the adjacent coils touch each other. In this case, the spring is completely compressed and no further compression is possible. The solid length is given by.

$$\text{Solid length} = N_t\, d$$

Where N_t = total number of coils

2) **Compressed length:** Compressed length is defined as the axial length of the spring that is subjected to maximum compressive force. In this case, the spring is subjected to maximum deflection. When the spring is subjected to maximum force, there should be some gap or clearance between the adjacent coils. The gap is essential to prevent clashing of the coils. The clashing allowance or the total axial gap is usually taken as 15% of the maximum deflection. Sometimes, an arbitrary decision is taken and it is assumed that there is a gap of 1 or 2 mm between adjacent coils under maximum load condition. In this case, the total axial gap is given by,

$$\text{Total gap} = (N_t - 1) \times \text{gap between adjacent coils}$$

3) **Free length:** Free length is defined as the axial length of an unloaded helical compression spring. In this case, no external force acts on the spring. Free length is an important dimension in spring design and manufacture. It is the length of the spring in free condition prior to assembly. Free length is given by,

$$\text{Free length} = \text{compressed length} + y = \text{solid length} + \text{total axial gap} + y$$

The **pitch** of the coil is defined as the axial distance between adjacent coils in uncompressed state of spring. It is denoted by **p**. It is given by,

$$\boldsymbol{p} = \textit{free length}/(N_t - 1)$$

The stiffness of the spring (**k**) is defined as the force required producing unit deflection. Therefore

$$\boldsymbol{k = p / \delta}$$

Where k= stiffness of the spring (N/mm)

p = axial spring force (N)

Y or $\boldsymbol{\delta}$ = axial deflection of the spring corresponding to force p (mm)

There are various names for stiffness of spring such as rate of spring, gradient of spring, scale of spring or simply spring constant. The stiffness of spring represents the slope of load deflection line. There are two terms are related to the spring coils, called active coils and inactive coils. **Active coils** are the coils in the spring, which contribute to spring action, support the external force and deflect under the action of force. A portion of the end coils, which is in contact with the seat, does not contribute to spring action and called **inactive coils**. These coils do not support the load and do not deflect under the action o external force. The number of inactive coils is given by,

$$\text{Inactive coils} = N_t - N$$

where N = number of active coils.

If we look at the free body diagram of the shaded region only (the cut section) then we shall see that at the cut section, vertical equilibrium of forces will give us force, F as indicated in the figure. This F is the shear force. The torque T, at the cut section and its direction is also marked in the figure. There is no horizontal force coming into the picture because externally there is no horizontal force present. So from the fundamental understanding of the free body diagram one can see that any section of the spring is experiencing a torque and a force. Shear force will always be associated with a bending moment. However, in an ideal situation, when force is acting at the centre of the circular spring and the coils of spring are almost parallel to each other, no bending moment would result at any section of the spring (no moment arm), except torsion and shear force. From the free body diagram, we have found out the direction of the internal torsion T and internal shear force F at the section due to the external load F acting at the centre of the coil. The cut sections of the spring, subjected to tensile and compressive loads respectively, are shown separately in the figure.

Stresses in Helical Springs

The following figure shows a round-wire helical compression spring loaded by the axial force *F*. We designate *D* as the *mean coil diameter* and *d* as the *wire diameter*. Now imagine that the spring is cut at some point (part b of Fig.), a portion of it removed, and the effect of the removed portion replaced by the net internal reactions. Then, as shown in the figure, from equilibrium the cut portion would contain a direct shear force *F* and a torsion

$$T = FD/2.$$

The maximum stress in the wire may be computed by superposition of the direct shear stress, with $V = F$ and the torsional shear stress. The result is

$$\tau_{max} = Tr/J + F/A$$

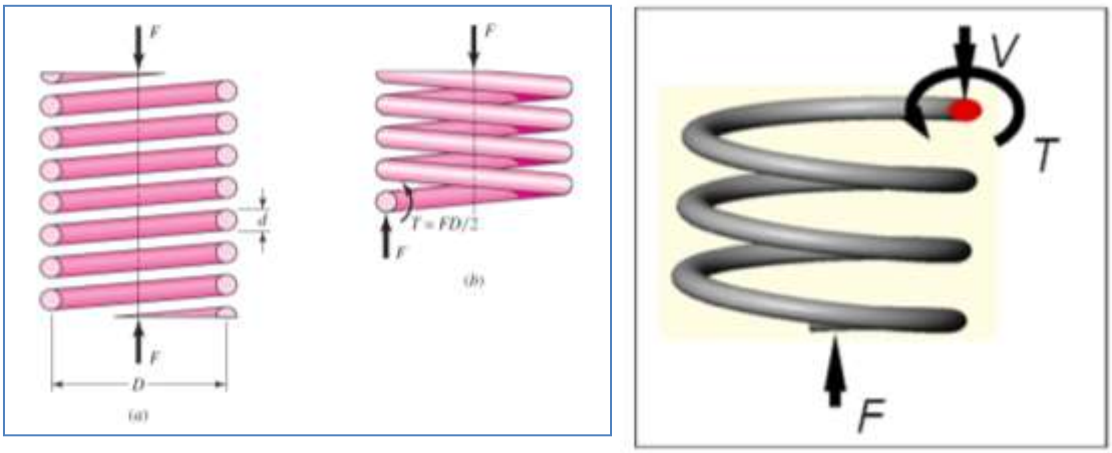
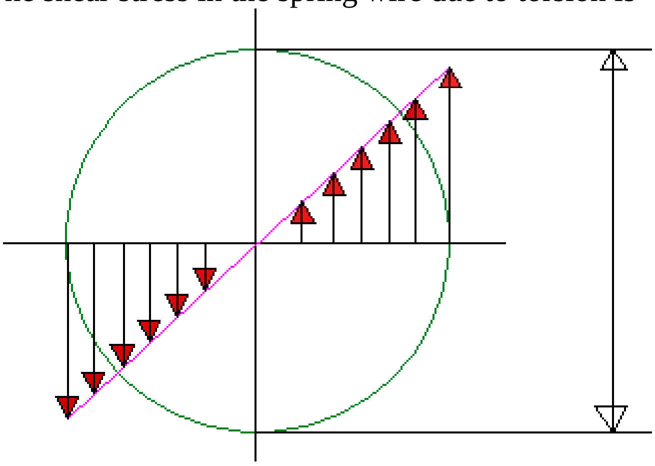
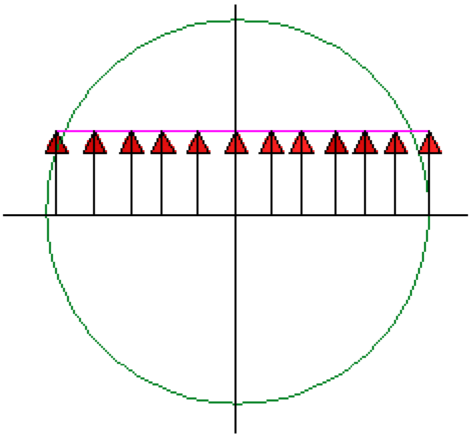


Figure: (*a*) Axially loaded helical spring; (*b*) free-body diagram showing that the wire is subjected to a direct shear and a torsional shear.

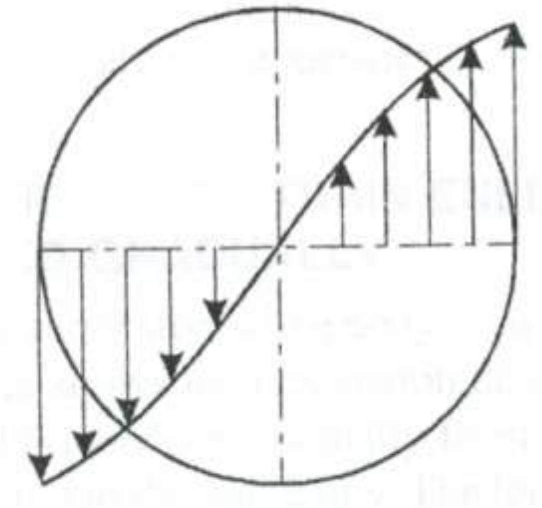
The shear stress in the spring wire due to torsion is



Average shear stress in the spring wire due to force F is



The above equation gives maximum shear stress occurring in a spring. K_s are the shear stress correction factor. The resultant diagram of torsional shear stress and direct shear stress is shown



From the above equation it can be observed that the effect of direct shear stress i.e.,

$$\tau = \frac{8\,F\,D}{\pi\,d^3} + \frac{4F}{\pi\,d^2} = K_s \frac{8\,F\,D}{\pi\,d^3}$$

where *Ks* is a *shear stress-correction factor* and is defined by the equation

$$K_s = \frac{2C + 1}{2\,C}$$

The use of square or rectangular wire is not recommended for springs unless space limitations make it necessary. Springs of special wire shapes are not made in large quantities, unlike those of round wire; they have not had the benefit of refining development and hence may not be as strong as springs made from round wire. When space is severely limited, the use of nested round-wire springs should always be considered. They may have an economical advantage over the special-section springs, as well as a strength advantage.

The Curvature Effect

The above equation is based on the wire being straight. However, the curvature of the wire increases the stress on the inside of the spring but decreases it only slightly on the outside. This curvature stress is primarily important in fatigue because the loads are lower and there is no opportunity for localized yielding. For static loading, these stresses can normally be neglected because of strain-strengthening with the first application of load. Unfortunately, it is necessary to find the curvature factor in a roundabout way. The reason for this is that the published equations also include the effect of the direct shear stress. Suppose Ks is replaced by another K factor, which corrects for both curvature and direct shear. Then this factor is given by either of the equations

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C} \quad or \quad K_B = \frac{4C+2}{4C-3}$$

The first of these is called the *Wahl factor*, and the second, the *Bergsträsser factor*. Since the results of these two equations differ by the order of 1 percent, the next equation is preferred. The curvature correction factor can now be obtained by canceling out the effect of the direct shear. Thus, the curvature correction factor is found to be

$$\boldsymbol{K_c = K_B / K_s = 2C(4C + 2)/(4C - 3)(2C + 1)}$$

Now, K_s , K_B or K_W , and K_c are simply stress-correction factors applied multiplicatively to *Tr/J* at the critical location to estimate a particular stress. There is *no* stress concentration factor. In this notes we will use

$$\tau = K_B \frac{8\,F\,D}{\pi\,d^3}$$

to predict the largest shear stress.

$$\tau = \frac{8FD}{\pi d^3}\frac{1}{2c}$$

Is appreciable for springs of small spring index ‘C’ Also the effect of wire curvature is neglected in equation (A)

Stresses in helical spring with curvature effect. What is curvature effect? Let us look at a small section of a circular spring, as shown in the figure. Suppose we hold the section b-c fixed and give a rotation to the section a-d in the anti clockwise direction as indicated

in the figure, then it is observed that line a-d rotates and it takes up another position, say a'-d'. Stresses in helical spring with curvature effect

What is curvature effect?

Let us look at a small section of a circular spring, as shown in the figure. Suppose we hold the

section b-c fixed and give a rotation to the section a-d in the anti clockwise direction as indicated

in the figure, then it is observed that line a-d rotates and it takes up another position, say a'-d'.

Deflection of Helical Springs

The deflection-force relations are quite easily obtained by using Castigliano’s theorem. The total strain energy for a helical spring is composed of a torsional component and a shear component. From previous equations the strain energy is

$$U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

Substituting $T = FD/2$, $l = \pi DN$, $J = \pi d^4/32$, and $A = \pi d^2/4$ results in

$$U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G}$$

where $N = N_a$ = number of active coils. Then using Castigliano's theorem to find total deflection y gives

$$y = \frac{\delta U}{\delta F} = \frac{8\,FD^3N}{d^4G} + \frac{4FDN}{d^2G}$$

Since $C = D/d$, this equation can be rearranged to yield

$$y = \frac{8\,FD^3N}{d^4G}\left(1 + \frac{1}{2C^2}\right) = \frac{8FD^3N}{d^4G}$$

The spring rate, also called the *scale* of the spring, is $k = F/y$, and so

$$k = \frac{d^4G}{8D^3N}$$

Compression Springs

The four types of ends generally used for compression springs are illustrated in following figure and table.

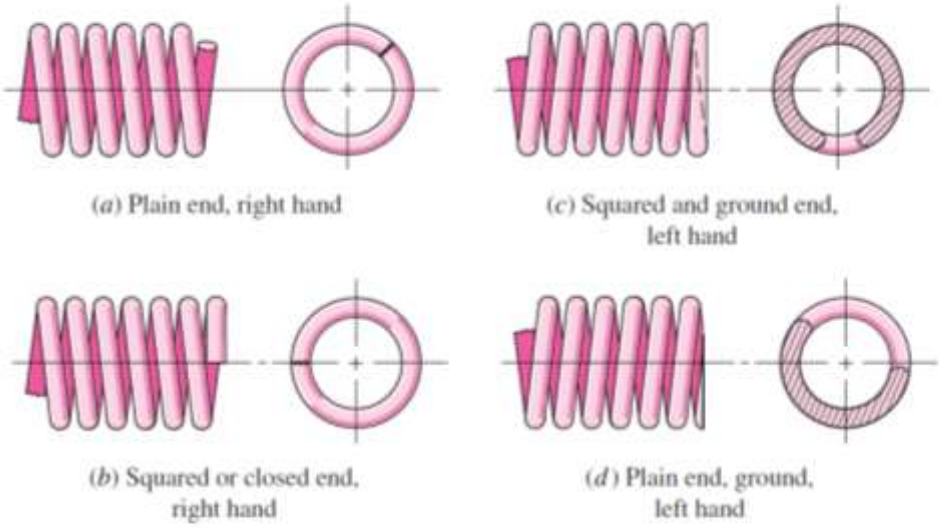


Figure : Types of ends for compression springs: (*a*) both ends plain; (*b*) both ends squared; (*c*) both ends squared and ground; (*d*) both ends plain and ground

A spring with *plain ends* has a noninterrupted helicoid; the ends are the same as if a long spring had been cut into sections. A spring with plain ends that are *squared* or *closed* is obtained by deforming the ends to a zero-degree helix angle. Springs should always be both squared and ground for important applications, because a better transfer of the load is obtained. The table shows how the type of end used affects the number of coils and the spring length.

Table : Formulas for the Dimensional Characteristics of Compression-Springs.(*Na* = Number of Active Coils) *Source:* From *Design Handbook*, 1987, p. 32. Courtesy of Associated Spring

Term	Type of Spring Ends			
	Plain	**Plain and Ground**	**Squared or Closed**	**Squared and Ground**
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/ N_a$	$L_0 /(N_a + 1)$	$(L_0 - 3d) /N_a$	$(L_0 - 2d)/N_a$

Note that the digits 0, 1, 2, and 3 appearing in the table are often used without question. *Some of these need closer scrutiny as they may not be integers.* This depends on how a springmaker forms the ends. Forys pointed out that squared and ground ends give a solid length $\boldsymbol{L_s}$ of

$$L_s = (N_t - a)d$$

where $\boldsymbol{a}$ varies, with an average of 0.75, so the entry $\boldsymbol{dN_t}$ in the table may be overstated. The way to check these variations is to take springs from a particular springmaker, close them solid, and measure the solid height. Another way is to look at the spring and count the wire diameters in the solid stack. *Set removal* or *presetting* is a process used in the manufacture of compression springs to induce useful residual stresses. It is done by making the spring longer than needed and then compressing it to its solid height. This operation *sets* the spring to the required final free length and, since the torsional yield strength has been exceeded, induces residual stresses opposite in direction to those induced in service. Springs to be preset should be designed so that 10 to 30 percent of the initial free length is removed during the operation. If the stress at the solid height is greater than 1.3 times the torsional yield strength, distortion may occur. If this stress is much less than 1.1 times, it is difficult to control the resulting free length. Set removal increases the strength of the spring and so is especially useful when the spring is used for energy-storage purposes. However, set removal should not be used when springs are subject to fatigue.

Stability

In previous section we learned that a column will buckle when the load becomes too large. Similarly, compression coil springs may buckle when the deflection becomes too large. The critical deflection is given by the equation

$$y_{cr} = L_0 C_1' \left[1 - \left(1 - \frac{C_2'}{\lambda_{eff}^2} \right)^{1/2} \right]$$

where y_{cr} is the deflection corresponding to the onset of instability. This equation is verified experimentally. The quantity λ_{eff} in this equation is the *effective slenderness ratio* and is given by the equation

$$\lambda_{eff} = \frac{\alpha L_0}{D}$$

C'_1 and C'_2 are elastic constants defined by the equations

$$C_1' = \frac{E}{2(E - G)}$$

$$C_2' = \frac{2\pi^2(E-G)}{2G+E}$$

One of these equations contains the *end-condition constant* $\boldsymbol{\alpha}$. This depends upon how the ends of the spring are supported. The following table gives values of $\boldsymbol{\alpha}$ for usual end conditions. Note how closely these resemble the end conditions for columns.

Table : End-Condition Constants α for Helical Compression Springs*

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed);other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

Absolute stability occurs when the term $\frac{C_2'}{\lambda_{eff}^2}$ is greater than unity. This means that the condition for absolute stability is that

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2}$$

For steels, this turns out to be

$$\boldsymbol{L_0 < 2.63\ D/\alpha}$$

For squared and ground ends $\boldsymbol{\alpha}$ = 0.5 and $\boldsymbol{L_0 < 5.26\ D}$.

Commonly used spring materials:

One of the important considerations in spring design is the choice of the spring material. Some of the common spring materials are given below.

Hard-drawn wire: This is cold drawn, cheapest spring steel. Normally used for low stress and static load. The material is not suitable at subzero temperatures or at temperatures above 1200C.

Oil-tempered wire: It is a cold drawn, quenched, tempered, and general purpose spring steel. It is not suitable for fatigue or sudden loads, at subzero temperatures and at temperatures above 1800C.

Chrome Vanadium: This alloy spring steel is used for high stress conditions and at high temperature up to 2200C. It is good for fatigue resistance and long endurance for shock and impact loads.

Chrome Silicon: This material can be used for highly stressed springs. It offers excellent service for long life, shock loading and for temperature up to 2500C.

Music wire: This spring material is most widely used for small springs. It is the toughest and has highest tensile strength and can withstand repeated loading at high stresses. It cannot be used at subzero temperatures or at temperatures above 1200C.

Stainless steel: Widely used alloy spring materials.

Phosphor Bronze / Spring Brass: It has good corrosion resistance and electrical conductivity. it is commonly used for contacts in electrical switches. Spring brass can be used at subzero temperatures.

Spring Manufacturing

Springs are manufactured either by hot- or cold-working processes, depending upon the size of the material, the spring index, and the properties desired. In general, prehardened wire should not be used if D/d < 4 or if d > 14 in. Winding of the spring induces residual stresses through bending, but these are normal to the direction of the torsional working stresses in a coil spring. Quite frequently in spring manufacture, they are relieved, after winding, by a mild thermal treatment. A great variety of spring materials are available to the designer, including plain carbon steels, alloy steels, and corrosion-resisting steels, as well as nonferrous materials such as phosphor bronze, spring brass, beryllium copper, and various nickel alloys. Descriptions of the most commonly used steels will be found in following table.

Table: High-Carbon and Alloy Spring Steels *Source:* From Harold C. R. Carlson, "Selection and Application of Spring Materials," *Mechanical Engineering*, vol. 78, 1956, pp. 331–334.

Name of Material	Similar Specifications	Description
Music wire, 0.80–0.95*C*	UNS G10850 AISI 1085 ASTM A228-51	This is the best, toughest, and most widely used of all spring materials for small springs. It has the highest tensile strength and can withstand higher stresses under repeated loading than any other spring material. Available in diameters 0.12 to 3mm(0.005 to0.125 in). Do not use above 120°C(250°F) or at subzero temperatures.
Oil-tempered wire, 0.60–0.70*C*	UNS G10650 AISI 1065 ASTM 229-41	This general-purpose spring steel is used for many types of coil springs where the cost of music wire is prohibitive and in sizes larger than available in music wire. Not for shock or impact loading. Available in diameters 3 to 12 mm (0.125 to0.5000 in), but larger and smaller sizes may be obtained. Not for use above 180°C (350°F) or at subzero temperatures.
Hard-drawn wire, 0.60–0.70*C*	UNS G10660 AISI 1066 ASTM A227-47	This is the cheapest general-purpose spring steel and should be used only where life, accuracy, and deflection are not too important. Available in diameters 0.8 to 12 mm (0.031 to 0.500 in). Not for use above 120°C (250°F) or at subzero temperatures.
Chrome-vanadium	UNS G61500 AISI 6150 ASTM 231-41	This is the most popular alloy spring steel for conditions involving higher stresses than can be used with the high-carbon steels and for use where fatigue resistance and long endurance are needed. Also good for shock and impact loads. Widely used for aircraft-engine valve springs and for temperatures to 220°C (425°F). Available in annealed or pre-tempered sizes 0.8 to 12 mm (0.031 to 0.500 in) in diameter.
Chrome-silicon	UNS G92540 AISI 9254	This alloy is an excellent material for highly stressed springs that require long life and are subjected to shock loading. Rockwell hardnesses of C50 to C53 are quite common, and the material may be used up to 250°C(475°F). Available from 0.8 to 12 mm (0.031 to 0.500 in) in diameter.

The UNS steels listed in Appendix A should be used in designing hot-worked, heavy-coil springs, as well as flat springs, leaf springs, and torsion bars.

Spring materials may be compared by an examination of their tensile strengths; these vary so much with wire size that they cannot be specified until the wire size is known. The material and its processing also, of course, have an effect on tensile strength. It turns out that the graph of tensile strength versus wire diameter is almost a straight line for some materials when plotted on log-log paper. Writing the equation of this line as

$$S_{ut} = A / d^m$$

furnishes a good means of estimating minimum tensile strengths when the intercept *A* and the slope *m* of the line are known. Values of these constants have been worked out from recent data and are given for strengths in units of kpsi and MPa in the following table. In the above equation when *d* is measured in millimeters, then *A* is in (MPa · mm^m) and when *d* is measured in inches, then *A* is in (kpsi · in^m).

Although the torsional yield strength is needed to design the spring and to analyze the performance, spring materials customarily are tested only for tensile strength perhaps because it is such an easy and economical test to make. A very rough estimate of the torsional yield strength can be obtained by assuming that the tensile yield strength is between 60 and 90 percent of the tensile strength. Then the distortion-energy theory can be employed to obtain the torsional yield strength ($S_{ys} = 0.577S_y$). This approach results in the range

$$\mathbf{0.35}S_{ut} \leq S_{sy} \leq \mathbf{0.52}S_{ut}$$

for steels.

For wires listed in the following table, the maximum allowable shear stress in a spring can be seen in column 3. Music wire and hard-drawn steel spring wire have a low end of range $S_{sy} = 0.45S_{ut}$. Valve spring wire, Cr-Va, Cr-Si, and other (not shown) hardened and tempered carbon and low-alloy steel wires as a group have

$$S_{sy} \geq 0.50S_{ut} .$$

Many nonferrous materials (not shown) as a group have $S_{sy} \geq 0.35S_{ut}$. In view of this, authors use the maximum allowable torsional stress for static application shown in the next table. For specific materials for which you have torsional yield information use this table as a guide. They provide set-removal information in the table, that $S_{sy} \geq 0.65S_{ut}$ increases strength through cold work, but at the cost of an additional operation by the spring maker. Sometimes the additional operation can be done by the manufacturer during assembly. Some correlations with carbon steel springs show that the tensile yield strength of spring wire in torsion can be estimated from $0.75Sut$. The corresponding estimate of the yield strength in shear based on distortion energy theory is

$$S_{sy} = 0.577(0.75)\ S_{ut} = 0.433\ S_{ut} .= 0.45\ S_{ut} .$$

Others discusses the problem of allowable stress and shows that

$$S_{sy} = \tau_{\text{all}} = 0.56S_{ut}$$

for high-tensile spring steels, which is close to the first value given for hardened alloy steels. They point out that this value of allowable stress is specified by Draft Standard 2089 of the German Federal Republic when shear formela is used without stress correction factor.

Table: Constants *A* and *m* of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires. *Source:* From *Design Handbook,* 1987, p. 19. Courtesy of Associated Spring.

Material	ASTM No.	Exponent m	Diameter, in	A, Kpsi.in^m	Diameter, mm	A, MPamm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

∗Surface is smooth, free of defects, and has a bright, lustrous finish.
†Has a slight heat-treating scale which must be removed before plating.
‡Surface is smooth and bright with no visible marks.
§Aircraft-quality tempered wire, can also be obtained annealed.
‖Tempered to Rockwell C49, but may be obtained untempered.
#Type 302 stainless steel.
∗∗Temper CA510.

Table : Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of S_{ut}		Diameter	*E*		*G*	
	Tension	Torsion	*d*, in	Mpsi	GPa	Mpsi	GPa
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD Spring A227	60–70	45–55	<0.032	28.8	198.6	11.7	80.7
			0.033–0.063	28.7	197.9	11.6	80.0
			0.064–0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85–90	45–50		28.5	196.5	11.2	77.2
Valve spring A230	85–90	50–60		29.5	203.4	11.2	77.2
Chrome-vanadium A231	88–93	65–75		29.5	203.4	11.2	77.2
A232	88–93			29.5	203.4	11.2	77.2
Chrome-silicon A401	85–93	65–75		29.5	203.4	11.2	77.2
Stainless steel A313*	65–75	45–55		28	193	10	69.0
17-7PH	75–80	55–60		29.5	208.4	11	75.8
414	65–70	42–55		29	200	11.2	77.2
431	65–75	45–55		29	200	11.2	77.2
420	72–76	50–55		30	206	11.5	79.3
Phosphor-bronze B159	75–80	45–50		15	103	4 6	41.4
Beryllium-copper B197	70	50		17	117.2	6.5	44.8
	75	50–55		19	131	7.3	50.3
Inconel alloy X-750	65–70	40–45		31	213.7	11.2	77.2

*Also includes 302, 304, and 316.
Note: See the next table for allowable torsional stress design values.

Table : Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications *Source:* Robert E. Joerres, "Springs," Chap. 6 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design*, 3rd ed., McGraw-Hill,New York, 2004.

Material	Maximum Percent of Tensile Strength	
	Before Set Removed (includes K_W or K_B)	After Set Removed (includes K_s)
Music wire and cold-drawn carbon steel	45	60–70
Hardened and tempered carbon and low-alloy steel	50	65–75
Austenitic stainless steels	35	55–65
Nonferrous alloys	35	55–65

Design Procedure:

The preferred range of spring index is $4 \le C \le 12$, with the lower indexes being more difficult to form (because of the danger of surface cracking) and springs with higher indexes tending to tangle often enough to require individual packing. This can be the first item of the design assessment.

The recommended range of active turns is $3 \le N_a \le 15$. To maintain linearity when a spring is about to close, it is necessary to avoid the gradual touching of coils (due to nonperfect pitch). A helical coil spring force-deflection characteristic is ideally linear. Practically, it is nearly so, but not at each end of the force-deflection curve. The spring force is not reproducible for very small deflections, and near closure, nonlinear behavior begins as the number of active turns diminishes as coils begin to touch. The designer confines the spring's operating point to the central 75 percent of the curve between no load, $F = 0$, and closure, $F = F_s$. Thus, the maximum operating force should be limited to $F_{\max} \le 7/8\ F_s$. Defining the fractional overrun to closure as ξ, where

$$F_s = (1 + \xi)F_{\max}$$

it follows that

$$F_s = (1 + \xi)F_{\max} = (1 + \xi)(7/8)\ F_s$$

From the outer equality $\xi = 1/7 \approx 0.15$. Thus, it is recommended that $\xi \ge 0.15$. In addition to the relationships and material properties for springs, we now have some recommended design conditions to follow, namely:

$$4 \le C \le 12$$

$$3 \le Na \le 15$$

$$\xi \ge 0.15$$

$$n_s \ge 1.2$$

where n_s is the factor of safety at closure (solid height).

When considering designing a spring for high volume production, the figure of merit can be the cost of the wire from which the spring is wound. The **fom** would be proportional to the relative material cost, weight density, and volume:

$$\text{fom} = -(\text{relative material cost})(\gamma\ \pi^2 d^2 N_t D/4)$$

For comparisons between steels, the specific weight γ can be omitted.

Spring design is an open-ended process. There are many decisions to be made, and many possible solution paths as well as solutions. In the past, charts, nomographs, and "spring design slide rules" were used by many to simplify the spring design problem. Today, the computer enables the designer to create programs in many different formats—direct programming, spreadsheet, MATLAB, etc. Commercial programs are also available. There are almost as many ways to create a spring-design program as there are programmers. Here, we will suggest one possible design approach.

Make the a priori decisions, with hard-drawn steel wire the first choice (relative material cost is 1.0). Choose a wire size d. With all decisions made, generate a column of parameters: d, D, C, OD or ID, N_a , L_s , L_0, $(L_0)_{cr}$, n_s, and fom. By incrementing wire sizes available, we can scan the table of parameters and apply the design recommendations by inspection. After wire sizes are eliminated, choose the spring design with the highest figure of merit. This will give the optimal design despite the presence of a discrete design variable d and aggregation of equality and inequality constraints. The procedure suggests to work in a table form like the one shown in the following figure and table. It is general enough to accommodate to the situations of as-wound and set-removed springs, operating over a rod, or in a hole free of rod or hole.

	d is chosen from the preferred wire sizes and then A can be calculated						
d							
A							
τ							
K_c							
C							
N_a							
D							
L_s							
L_0							
$(L_0)_{cr}$							
n_s							
fom							

Figure: The Design Procedure for the Helical Spring

Table: Spring's wire preferred diameters (Metric sizes).

Preferred metric diameters **(d)***	13.0, 12.0, 11.0, 10.0, 9.0, 8.5, 8.0, 7.0, 6.5, 6.0, 5.5, 5.0, 4.8, 4.5, 4.0, 3.8, 3.5, 3.0, 2.8, 2.5, 2.0, 1.8, 1.6, 1.4, 1.2, 1.0, 0.90, 0.80, 0.70, 0.65, 0.60 or 0.55, 0.50 or 0.55, 0.45, 0.45, 0.40, 0.40, 0.35, 0.35, 0.30 or 0.35, 0.30, 0.28, 0.25, 0.22, 0.22, 0.20, 0.20, 0.18

*The preferred metric sizes are from Associated Spring. Barnes Group, Inc., and are listed as the nearest preferred metric size to the U.S. Steel Wire Gage. The gage numbers do not apply.

Now examine the table and perform the adequacy assessment. The shading of the table indicates values outside the range of recommended or specified values. The spring index constraint $4 \leq C \leq 12$ rules out diameters that make C out of range. The constraint $3 \leq N_a \leq 15$ rules out wire diameters that result in values out of the range. The $L_s \leq 1$ constraint rules out diameters less than 0.080 in. The $L_0 \leq 4$ constraint rules out other diameters. The buckling criterion rules out free lengths longer than $(L_0)_{cr}$, which rules out more diameters. The factor of safety n_s is exactly 1.20 because the mathematics forced it. Had the spring been in a hole or over a rod, the helix diameter would be chosen without reference to $(n_s)d$. The result is that there are only few springs. The figure of merit decides and the decision is the design with the best of them.

Critical Frequency of Helical Springs

If a wave is created by a disturbance at one end of a swimming pool, this wave will travel down the length of the pool, be reflected back at the far end, and continue in this back-and-forth motion until it is finally damped out. The same effect occurs in helical springs, and it is called *spring surge*. If one end of a compression spring is held against a flat surface and the other end is disturbed, a compression wave is created that travels back and forth from one end to the other exactly like the swimming-pool wave. Spring manufacturers have taken slow-motion movies of automotive valve-spring surge. These pictures show a very violent surging, with the spring actually jumping out of contact with the end plates. When helical springs are used in applications requiring a rapid reciprocating motion, the designer must be certain that the physical dimensions of the spring are not such as to create a natural vibratory frequency close to the frequency of the applied force; otherwise, resonance may occur, resulting in damaging stresses, since the internal damping of spring materials is quite low.

The governing equation for the translational vibration of a spring is the wave equation. The harmonic, *natural*, frequencies for a spring placed between two flat and parallel plates, in radians per second, are

$$\omega = m\,\pi\sqrt{\frac{kg}{W}} \qquad m = 1,2,3, \ldots.$$

where the fundamental frequency is found for $m = 1$, the second harmonic for $m = 2$, and so on. We are usually interested in the frequency in cycles per second; since $\omega = 2\,\pi\,f$, we have, for the fundamental frequency in hertz,

$$f = \frac{1}{2}\sqrt{\frac{kg}{W}}$$

assuming the spring ends are always in contact with the plates.

Studies show that the frequency is

$$f = \frac{1}{4}\sqrt{\frac{kg}{W}}$$

where the spring has one end against a flat plate and the other end free. They also point out that this equation applies when one end is against a flat plate and the other end is driven with a sine-wave motion.

The weight of the active part of a helical spring is

$$W = AL\gamma = \frac{\pi d^2}{4}\,(\pi D N_a)(\gamma) = \frac{\pi^2 D N_a \gamma}{4}$$

where γ is the specific weight.

The fundamental critical frequency should be greater than 15 to 20 times the frequency of the force or motion of the spring in order to avoid resonance with the harmonics. If the frequency is not high enough, the spring should be redesigned to increase *k* or decrease *W*.

Threads

The helical-thread screw was undoubtedly an extremely important mechanical invention. It is the basis of power screws, which change angular motion to linear motion to transmit power or to develop large forces (presses, jacks, etc.), and threaded fasteners, an important element in nonpermanent joints.

Screw threads serve three basic functions in mechanical systems;

1) to provide a clamping force

2) to restrict or control motion, and

3) to transmit power.

Geometrically, a screw thread is a helical incline plane. A helix is the curve defined by moving a point with uniform angular and linear velocity around an axis. The distance the point moves linear (parallel to the axis) in one revolution is referred to as pitch or lead. The term "***internal threads"*** refers to threads cut into the sidewall of an existing hole. ***External threads*** refers to threads cut or rolled into the external cylindrical surface of a fastener or stud. The size most commonly associated with screw threads is the ***nominal diameter***. Nominal diameter is a more of a label than a size. For example, a bolt and nut may be described as being (**M16**) diameter. But neither the external threads of the bolt nor the internal threads of the nut are exactly 16 mm diameter. In fact, the bolt diameter is a little smaller and the nut diameter a little larger. But it is easier to specify the components by a single size designation since the bolt and nut are mating components.

Thread Joint can be classified as part of the temporary fasteners as shown in the following figure.

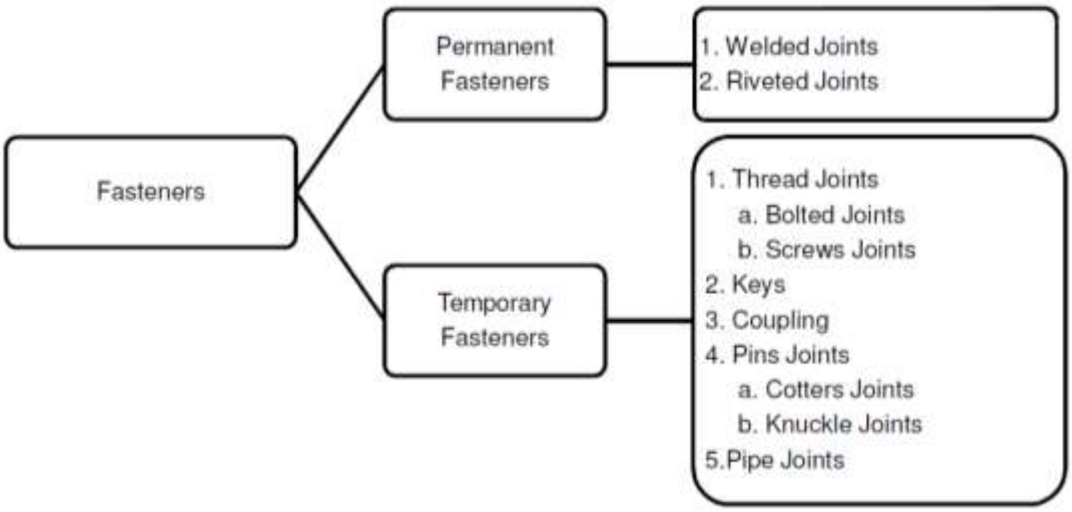


Figure Types of Fasteners.

Definitions and Threads Terminology:

Bolts: They are basically threaded fasteners normally used with nuts.

Screws: They engage either with preformed or self-made internal threads.

Studs: They are externally threaded headless fasteners. One end usually meets a tapped component and the other with a standard nut.

There are different forms of bolt and screw heads for a different usage. These include bolt heads of square, hexagonal or eye shape and screw heads of hexagonal, Fillister, button head, counter sunk or Phillips type. These are shown in the following figures.

Crest: the peak of the thread for external threads, the valley of the thread for internal threads.

Major Diameter [d]: The largest diameter of a screw thread.

Minor Diameter (or root) [d_r]: The smallest diameter of a screw thread.
Pitch Diameter [d_p]: nominally the mean of the major and minor diameters.
Thread Angle : The included angle between two adjacent thread walls.
Pitch : The distance measured axially, between corresponding points on the consecutive thread forms in the same axial plane and on the same side of axis is known as pitch length. The pitch in U.S. units is the reciprocal of the number of thread forms per inch *n*.
Lead : It is axial distance a screw thread advances in one revolution. For a single thread the lead is the same as the pitch. A *multiple-threaded* product is one having two or more threads cut beside each other (imagine two or more strings wound side by side around a pencil). Standardized products such as screws, bolts, and nuts all have single threads; a *double-threaded* screw has a lead equal to twice the pitch, a *triple-threaded* screw has a lead equal to 3 times the pitch, and so on. All threads are made according to the *right-hand rule* unless otherwise noted. That is, if the bolt is turned clockwise, the bolt advances toward the nut.
Square Thread : A square thread is formed if the generating plane section is square.
Acme Thread : The vee thread is created by a triangular section while trapezoidal thread has a trapezium section. This thread is also known as the Acme thread.

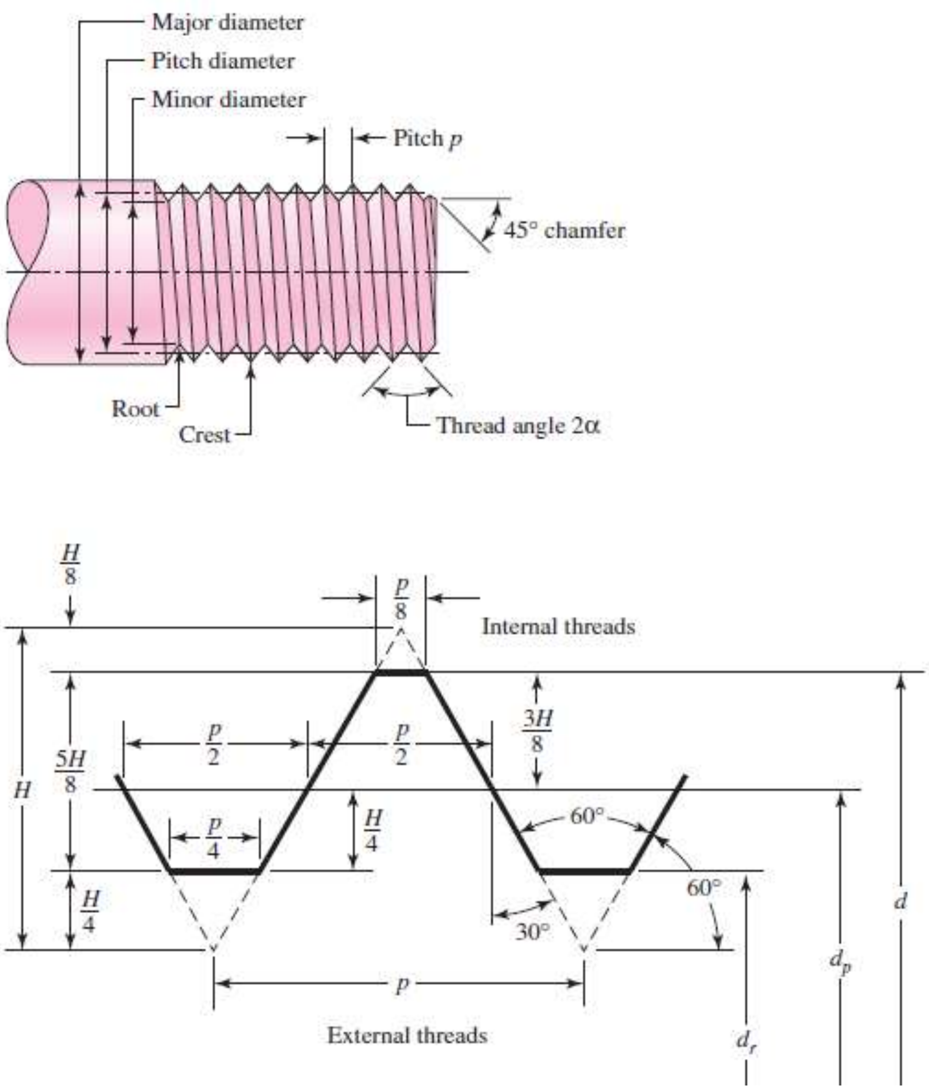


Figure: Thread Terminology and Profile [after Shigley, 2011]

Threaded Fastener

Apart from transmitting motion and power the threaded members are also used for fastening or jointing two elements. The threads used in power screw are square or Acme while threads used in fastening screws have a Vee profile. Because of large transverse inclination the effective friction coefficient between the screw and nut increases by equation($\bar{f} = f / \cos\theta$) where f is the basic coefficient of friction of the pair of screw and nut, θ is the half of thread angle and $\bar{f}$ is the effective coefficient of friction. The wedging effect of transverse inclination of the thread surface was explained before. According to IS : 1362-1962 the metric thread has a thread angle of 60^o. The other proportions of thread profile are shown in the above figure. IS : 1362 designates threads by ***M*** followed by a figure representing the major diameter, ***d***. For example a screw or bolt having the major diameter of 2.5 mm will be designated as ***M* 2.5**. The standard describes the major (also called nominal) diameter of the bolt and nut, pitch, pitch diameter, minor or core diameter, depth of bolt thread and area resisting load (Also called stress area). Pitch diameter in case of V-threads corresponds to mean diameter in square or Acme thread. The following two tables describes V-thread dimensions according to IS : 1362.

Table: Dimensions of V-threads (Coarse) [IS : 1362]

Designation	p (mm)	d or D (mm)	D_p (mm)	D_C (mm)		Thread Depth (mm)	Stress Area (mm^2)
				Nut	Bolt		
M 0.4	0.1	0.400	0.335	0.292	0.277	0.061	0.074
M 0.8	0.2	0.800	0.670	0.584	0.555	0.123	0.295
M 1	0.25	1.000	0.838	0.729	0.693	0.153	0.460
M 1.4	0.3	1.400	1.205	1.075	1.032	0.184	0.983
M 1.8	0.35	1.800	1.573	1.421	1.371	0.215	1.70
M 2	0.4	2.000	1.740	1.567	1.509	0.245	2.07
M 2.5	0.45	2.500	2.208	2.013	1.948	0.276	2.48
M 3	0.5	3.000	2.675	2.459	2.387	0.307	5.03
M 3.5	0.6	3.500	3.110	2.850	2.764	0.368	6.78
M 4	0.7	4.000	3.545	3.242	3.141	0.429	8.78
M 5	0.8	5.000	4.480	4.134	4.019	0.491	14.20
M6	1	6.000	5.350	4.918	4.773	0.613	20.10
M 8	1.25	8.000	7.188	6.647	6.466	0.767	36.60
M 10	1.5	10.000	9.026	8.876	8.160	0.920	58.30
M 12	1.75	12.000	10.863	10.106	9.858	1.074	84.00
M 14	2	14.000	12.701	11.835	11.564	1.227	115.00
M 16	2	16.000	14.701	13.898	13.545	1.227	157.00
M 18	2.5	18.000	16.376	15.294	14.933	1.534	192
M 20	2.5	20.000	18.376	17.294	16.933	1.534	245
M 24	3	24.000	22.051	20.752	20.320	1.840	353
M 30	3.5	30.000	27.727	26.211	25.706	2.147	561
M 36	4	36.000	33.402	31.670	31.093	2.454	976
M 45	4.5	45.000	42.077	40.129	39.416	2.760	1300
M 52	5	52.000	48.752	46.587	45.795	3.067	1755
M 60	5.5	60.000	56.428	54.046	53.177	3.374	2360

Table: Dimensions of V-threads (Fine) [IS : 1362]

Designation	*p* (mm)	*d* or *D* (mm)	d_p (mm)	D_C (mm)		Thread Depth (mm)	Stress Area (mm^2)
				Nut	Screw		
M 8 × 1	1	8.000	7.350	6.918	6.773	0.613	39.2
M 10 × 1.25	1.25	10.000	9.188	8.647	8.466	0.767	61.6
M 12 × 1.25	1.25	12.000	11.184	10.647	10.466	0.767	92.1
M 14 × 1.5	1.5	14.000	13.026	12.376	12.166	0.920	125
M 16 × 1.5	1.5	16.000	15.026	14.376	14.160	0.920	167
M 18 × 1.5	1.5	18.000	17.026	16.376	16.160	0.920	216
M 20 × 1.5	1.5	20.000	19.026	18.376	18.160	0.920	272
M 22 × 1.5	1.5	22.000	21.026	20.376	20.160	0.920	333
M 24 × 2	2	24.000	22.701	21.835	24.546	1.227	384
M 27 × 2	2	27.000	25.701	24.835	24.546	1.227	496
M 30 × 2	2	30.000	28.701	27.835	27.546	1.227	621
M 33 ×2	2	33.000	31.701	30.335	30.546	1.227	761
M 36 × 3	3	36.000	34.051	32.752	32.391	1.840	865
M 39 × 3	3	39.000	37.051	35.752	35.391	1.840	1028

Wide variety of threaded fasteners are used in engineering practice. These are cylindrical bars, which are threaded to screw into nuts or internally threaded holes. The following figure depicts three commonly used fasteners. A bolt has a head at one end of cylindrical body. The head is hexagonal in shape. The other end of the bolt is threaded. The bolt passes through slightly larger holes in two parts and is rotated into hexagonal nut, which may sit on a circular washer. The bolt is rotated into the nut by wrench on bolt head.

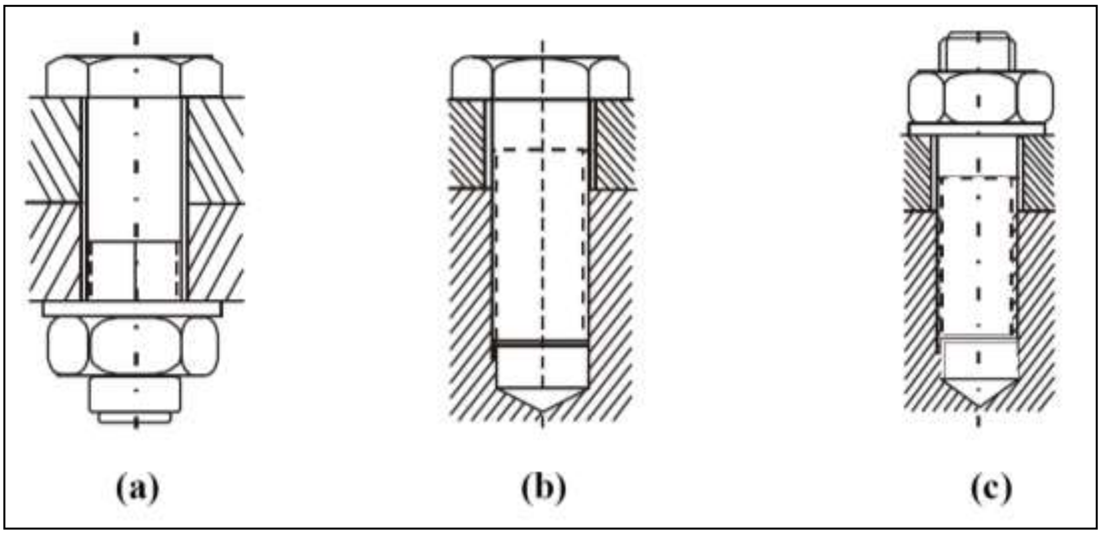


Figure: Three Types of Threaded Fasteners, (a) two parts are clamped between bolt head and nut (b) A screw with a head and threads on part of its cylindrical body threaded into an internally threaded hole. (c) A stud is threaded at both ends and does not have a head. One of its end screws into threaded hole while the other threaded end receives nut.

The bolts are available as ready to use elements in the market. Depending upon manufacturing method they are identified as black, semi finished or finished. The head in black bolt is made by hot heading. The bearing surfaces of head or shank are machine finished and threads are either cut or rolled. In semi finished bolts the head is made by cold or hot heading. The bearing surfaces of head or shank are machine finished and threads are either cut or rolled. A finished bolt is obtained by machining a bar of same section as the head. The threads are cut on a turret lathe or automatic thread cutting machine.

Besides hexagonal head the bolt or screw may have shapes as shown in the following figure, except the hexagonal and square head which are common in bolts, other forms are used in machine screws. Those at (a) and (b) are tightened with wrench, the bolt or screw with internal socket is rotated with a hexagonal key, at (c) and the screws carrying slits in the head are rotated with screw driver.

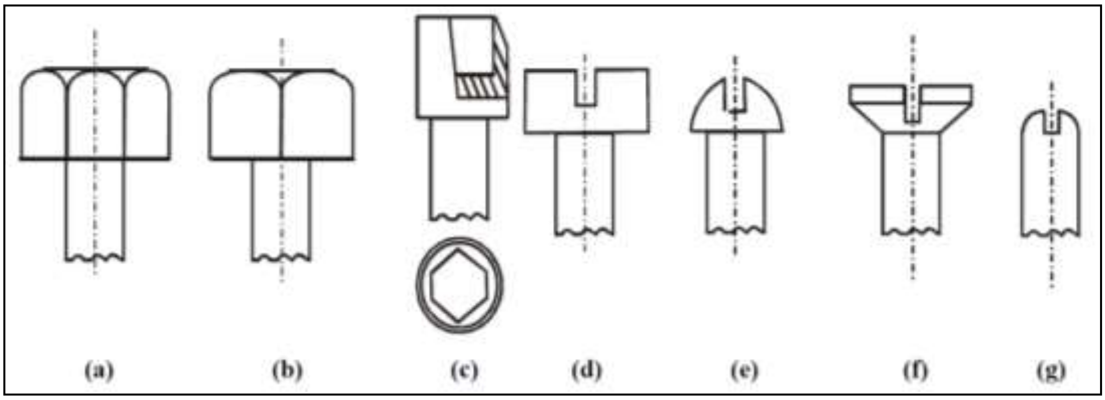


Figure: Heads of Threaded Fasteners; (a) Hexagonal; (b) Square; (c) Internal Socket; (d) Circular with a Slit; (e) Button with Slit; (f) Counter Sunk with a Silt; (g) Plain with a Slit

Effect of thread angle on strength:

The lower the value of the thread angle, the greater the load carrying capability of the thread. The force of mating threads is normal to the surface of the thread. This is shown in the following figure as F. The components of the force F transverse and parallel to the axis are shown as F_t and F_a. The component of force typically responsible for failure is that applied transverse to the axis of the thread. It is this load that can cause cracking in internal threads, especially under cyclic loads. Internal threads are more susceptible since they are typically cut and cutting operations in metals produce surface irregularities that can contribute to crack growth. External threads are typically rolled onto a fastener and therefore lack the surface flaws of cut threads. As the thread angle decreases, the component F_t gets smaller. This is why square and buttress threads are usually used for power transfer applications.

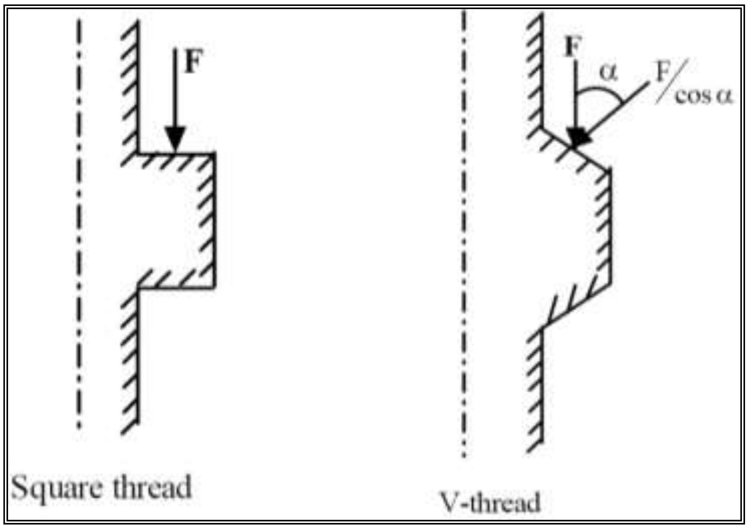


Figure Thread Forces

Failure Of Bolts And Screws

The bolts and screws may fail because of following reasons :

(a) Breaking of bolt shank
(b) Stripping of threads
(c) Crushing of threads
(d) Bending of threads
Invariably when bolt is tightened it is subjected to tensile load along its axis. There may be rare occasion where bolt is pretentioned. The bolt loading situations may be identified as :
(a) No initial tension, bolt loaded during operation.
(b) Only initial tension and no loading afterwards.
(c) After initial tension bolt is further loaded in tension during operation.
(d) In addition to loading initially bolt may be subjected to bending moment and/or shearing forces.
(e) In eccentrically loaded bolted joint, the bolts are subjected to shearing stress which is dominant. Initial tension is additional. We will analyze this problem as riveted joint.

Permissible Stresses In Bolts
Bolts are often made in steel having carbon percentage varying between 0.08 to 0.25. However, high quality bolts and particularly those of smaller diameter are made in alloy steel and given treatment of quenching followed by tempering. Medium carbon steels may also be improved in tensile strength by similar heat treatment. Since it is not always possible to determine the wrench torque when bolts are fitted on shop floor, the initial tightening torque often tends to be higher than necessary. This will obviously induce higher stress in the bolt even without external load. Such stresses are particularly high in case of smaller diameter bolt and reduce as the bolt diameter increases. This kind of tightening stresses call for varying permissible stresses in case of bolt which are small when bolt diameter is small and high when bolt diameter is large. This is unlike other machine parts where trend of permissible stress is just the reverse. The correlation of permissible stress and bolt diameter requires that the process of selection of diameter will be reiterative. An empirical formula that correlates the permissible tensile stress, σ_{tp}, and bolt diameter at the stress section d_1, is usually, used and this formula is plotted in the following Figure which can be used as an alternative to formula. Both the formula and the Figure are applicable to medium carbon steel bolts only.

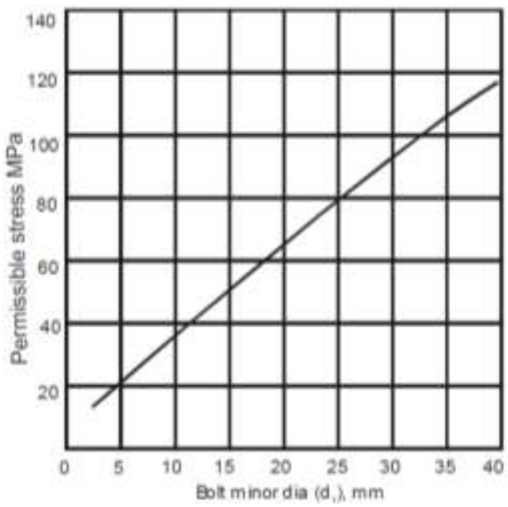


Figure: Permissible Bolt Stress as Function of Minor (Core) Diameter for Medium Carbon Steel Bolts
The student must see that the core section in V-thread means the same thing as in case of square thread. It is the core section, which carries the stress and is identified by core diameter d_1. This diameter can be seen in the figures and tables. The **Standard** tables also describe the area of the core section under the heading of stress area. The design equation for the bolt or screw is same with the difference that the fastener will always be in tension. So, if the permissible tensile stress is σ_{tp}, then

$$\sigma_{tp} = \frac{4 \times 1.3\, F}{\pi d_1^2} = \frac{5.2\, W}{\pi d_1^2}$$

Threads Production:
Threads are produced in various ways as follows:
1. Cutting Process.
2. Forming process.
3. Grinding process.

Standardization of Threads: (Standard Inch Units)
To facilitate their use, screw threads have been standardized. In 1948, the United States, Great Britain and Canada established the current system for standard inch dimension threads. This is the Unified thread series and consists of specifications for Unified Coarse (UNC) Unified Fine (UNF) and Unified Extra Fine (UNEF) threads. Metric threads are also standardized. Metric thread specification is given through ISO standards. Thread information is available in tabular form from many sources including Mechanical Drawing texts and Machine Design handbooks.
Thread form:
Thread form is a classification based upon the cross-sectional profile of the thread. The standard thread form for inch unit threads in U.S. is the *Unified* (UN) thread form. This thread form is characterized by a 60 degree thread angle and a flat crest and rounded root.

Thread series:
Thread series is a standard based upon the number of threads/inch for a specific nominal diameter. Standards for standard inch units are:
Coarse (C), *Fine* (F), *Extra-Fine* (EF). The figure at right shows fine and coarse thread fasteners. The designation is based upon the number of threads per unit length. A short discussion of each thread series is given below.

Figure: Photograph of Course and Fine Threads

Threads per Inch:
Literally a measure of the number of crests per unit of length measured along the axis of the thread. The number of threads/inch for a thread series is given by standard and may be found in thread tables. The Tap Chart shown later in this document gives the number of threads/inch based upon threads series and nominal diameter.
Descriptions of the Thread Series:
Unified Coarse. UNC is the most commonly used thread on general-purpose fasteners. Coarse threads are deeper than fine threads and are easier to assemble without cross threading. UNC threads are normally easier to remove when corroded, owing to their sloppy fit. A UNC fastener can be procured with a class 3 (tighter) fit if needed (fit classes covered below).

Unified Fine. UNF thread has a larger minor diameter than UNC thread, which gives UNF fasteners slightly higher load-carrying (in shear) and better torque-locking capabilities than UNC fasteners of the same material and outside diameter. The fine threads have tighter manufacturing tolerances than UNC threads, and the smaller lead angle allows for finer tension adjustment. UNF threads are frequently used in cases where thread engagement is minimized due to thinner wall thickness.

Unified national extra fine. UNEF is a thread finer than UNF and is common to the aerospace field. This thread is particularly advantageous for tapped holes in hard materials as well as for tapped holes in thin materials where engagement is at a minimum.

Class fit:

Class fit is a specification of how tightly mating external and internal threads will mesh. It is based upon the difference in the values of the respective pitch diameters. These differences are in the thousandths of an inch. For the Unified thread form, the classes of fit are:

Class 1: Loose fit. Threads may be assembled easily by hand. Used in cases where frequent assembly/disassembly required. Typically require use of locking devices such as lock washers, locking nuts, jam nuts, etc. Class 1 fits are common for bolts and nuts.

Class 2: Standard fit. Threads may be assembled partly by hand. Most common fit in use. Used in semi-permanent assemblies.

Class 3: Tight fit. Can be started by hand, but requires assistance (tools) to advance threads. Common for set screws. Used in permanent assemblies.

An additional designation is made for external (A) versus internal (B) threads and is included as a postscript to the numerical designation.

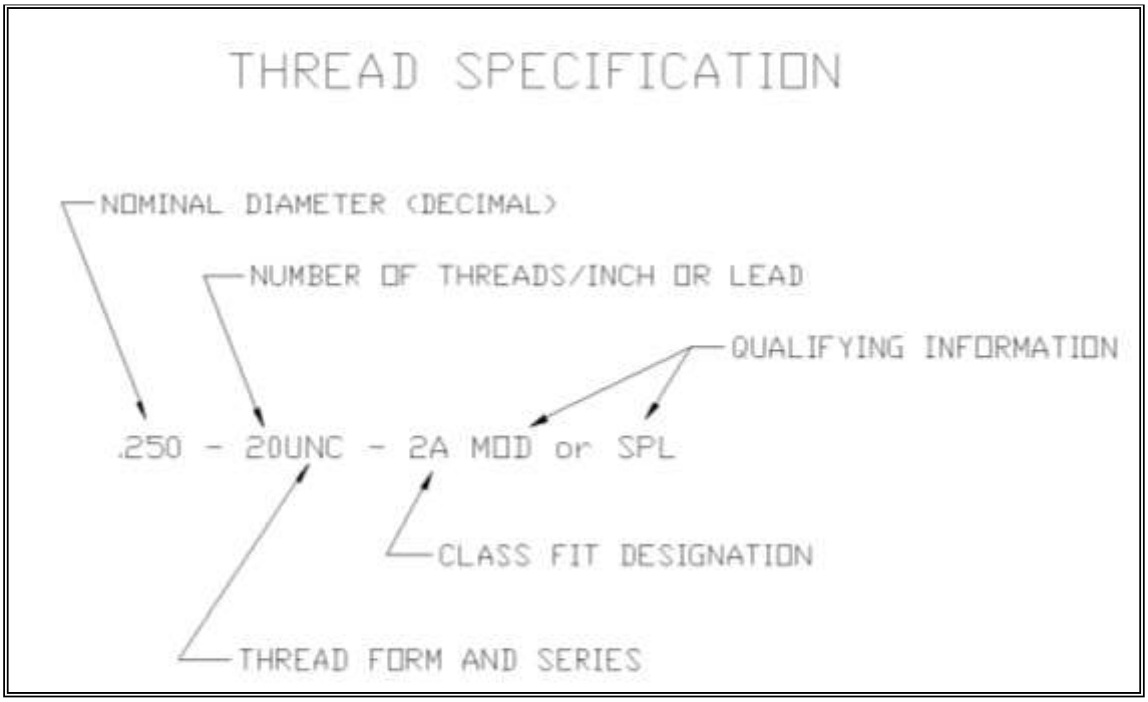


Figure: Thread designation **(Standard Inch Units)**

Standard inch unit thread specification examples

.4375 - 20UNF - 2A, LH

.500 - 13UNC – 1A

.375 - 24UNEF - 2B

Specification of Metric Threads:

Metric threads are defined in the standards document ISO 965-1. Metric thread specifications always begin with thread series designation (for example M or MJ), followed by the fastener's nominal diameter and thread pitch (both in units of millimeters) separated by the symbol "x".

Metric thread specification examples

MJ6 x 1 - 4H5H

M8 x 1.25 - 4h6h LH (i.e. Left hand thread)

M10 x 1.5 - 4h5h

Metric thread series:

There exists multiple metric thread series used for special applications. The standard is the M series. The MJ series is one of the most common of the special application threads.

M Series: Standard metric thread profile

MJ Series: Modified series in which crest and root radii are specified

Metric thread fits:

A fit between metric threads is indicated by internal thread class fit followed by external thread tolerance class separated by a slash; e.g., M10 x 1.5-6H/6g. The class fit is specified by tolerance grade (numeral) and by tolerance position (letter).

General purpose fit

6g (external) 6H (internal)

Close fit

5g6g (external) 6H (internal)

If thread fit designation (e.g., "-6g") is omitted (e.g., M10 x 1.5), it specifies a **"medium" fit**, which is 6H/6g. The 6H/6g fit is the standard ISO tolerance class for general use.

English unit internal and external thread class fit 2B/2A is essentially equivalent to ISO thread class fit 6H/6g. English unit class fit 3B/3A is approximately equivalent to ISO class fit 4H5H/4h6h.

Default metric fastener thread pitch. If metric thread pitch designation (e.g., " x 1.5") is omitted, it specifies coarse pitch threads. For example, M10 or M10-6g, by default, specifies M10 x 1.5. The standard metric fastener thread series for general purpose threaded components is the M thread profile and the coarse pitch thread series.

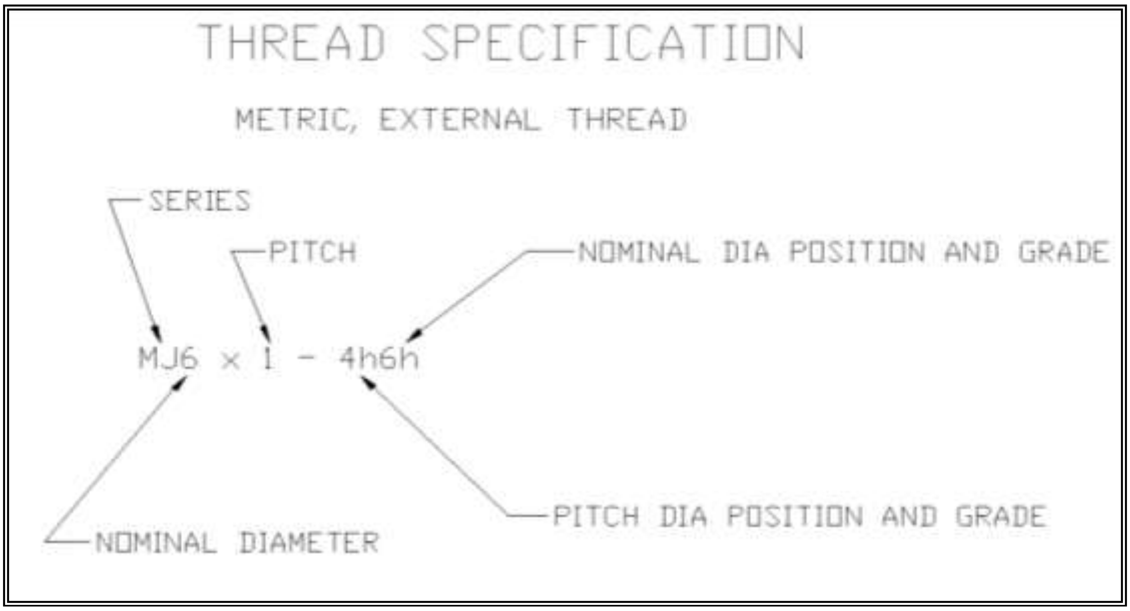


Figure: Thread designation **(Standard Metric Units)**

Metric fastener thread series compatibility. Metric fastener thread series M is the common thread profile. Thread series MJ designates the *external* thread has an increased root radius (shallower root relative to external M thread profile), thereby having higher fatigue strength (due to reduced stress concentrations), but requires the truncated crest height of the MJ internal thread to prevent interference at the external MJ thread root. M external threads are compatible with both M *and* MJ internal threads.

M10 x 1.5-6g means metric fastener thread series M, fastener nominal size (nominal major diameter) 10 mm, thread pitch 1.5 mm, *external* thread class fit 6g. If referring to *internal* thread tolerance, the "g" would be uppercase.

Left Hand Threads:

Unless otherwise specified, screw threads are assumed to be right-handed. This means that the direction of the thread helix is such that a clockwise rotation of the thread will cause it to advance along its axis. Left-handed threads advance when rotated counter clockwise. Left-handed threads are often used in situations where rotation loads would cause right-hand threads to loosen during service. A common example is the bicycle. The pedals of a bicycle are attached to the crank arm using screw threads. The pedal on one side of the bicycle uses right-hand threads and the other uses left-hand. This prevents the motion of pedals and crank from unscrewing the pedal and having it fall off during use. Left-hand threads must be indicated in the thread specification. This is accomplished by appending “LH” to the end of the specification.

Local Notes

Local notes, also referred to as callouts, are included on a drawing to specify information for a specific feature of a component or assembly. The feature being referenced is indicated through the use of a leader line. The leader line points to the feature in question and terminates at the note. One common example of a local note is the specification of the size dimension of a hole feature. When a callout is made to a hole feature, the leader line should reference the circular view of hole with line pointing toward the center of the circle. The note should be written in the order of operations performed. (e.g. drill then thread) and the leader arrowhead should touch the representation of the last operation performed.

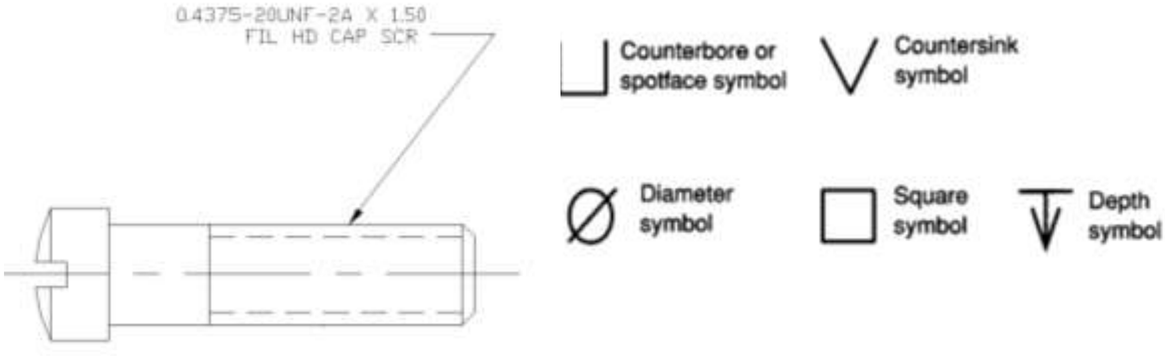


Figure: Callout Examples and Common Callout Symbols

The two examples of callouts below reference counterbored and countersunk holes. In case you have forgotten, counterboring and countersinking are secondary machining operations used to create cylindrical and conical (respectively) enlargements of a hole, usually for the purpose of recessing a fastener head.
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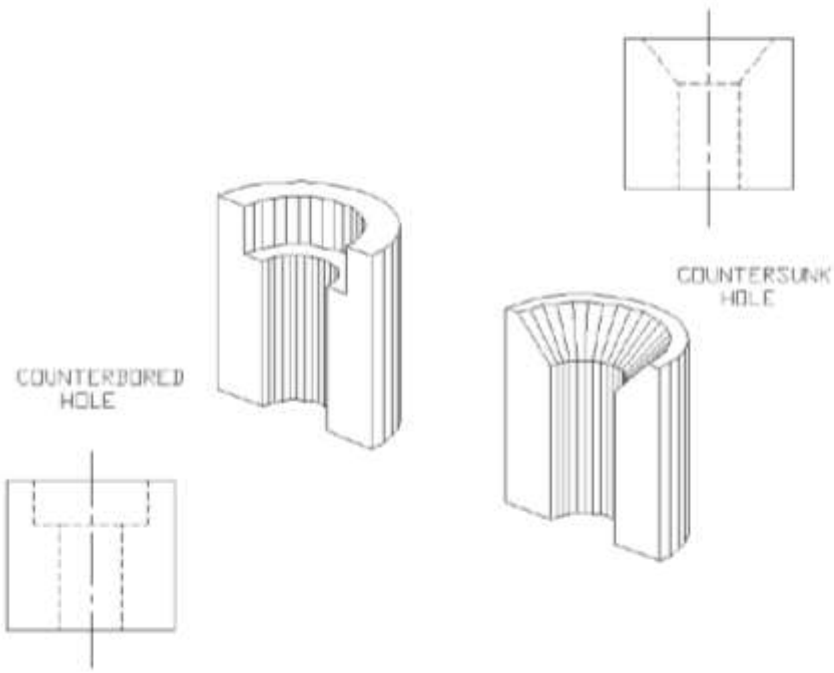


Figure: Counterbored and Countersunk Holes

In the examples shown at right the pilot hole is specified first then the counterbore or countersink is specified. Notice that no specification of operation is given for the pilot hole. Operation specifications such as “DRILL” or “BORE” are no longer included in notes and callouts. Rather only the feature sizes (and tolerances, if applicable) are included.

Counterbore specification:

Include the diameter of the counterbore, which is based upon fastener head diameter with a clearance value added. (Refer to Head Dimension Tables) Include the depth of the counterbore, which is based upon head profile height. (Refer to Head Dimension Tables for this information)

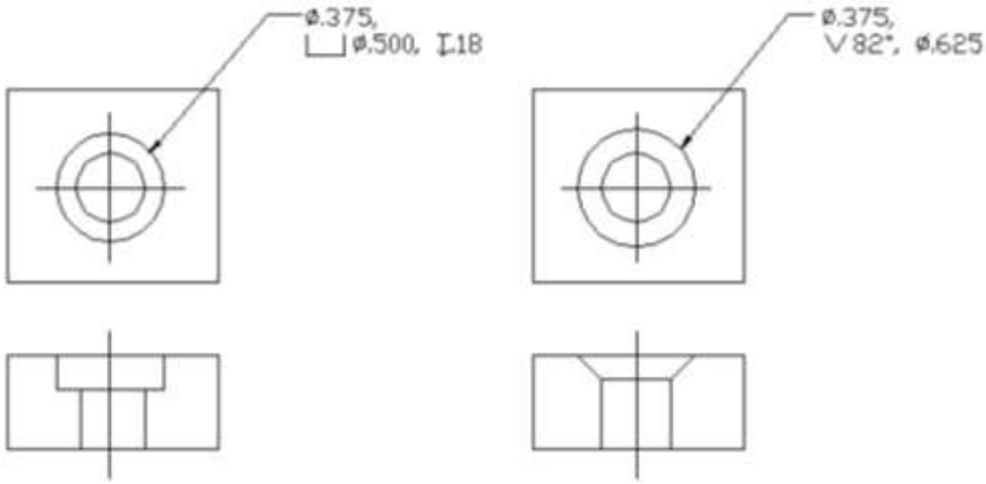


Figure: Counterbore and Countersink Callouts

Countersink specification:

Include the angle of countersink and either;
1) depth of countersink or
2) diameter of maximum opening (based upon fastener head diameter plus 1/64 typ. or equivalent)
Examples of metric notes for counterbored, countersunk and spot-faced holes are given at right. The depth of a machined hole is categorized as being either thru or blind. A thru hole begins at the penetrating surface and terminates at another surface. Therefore the "depth" of the hole is based upon the distance between the two surfaces.

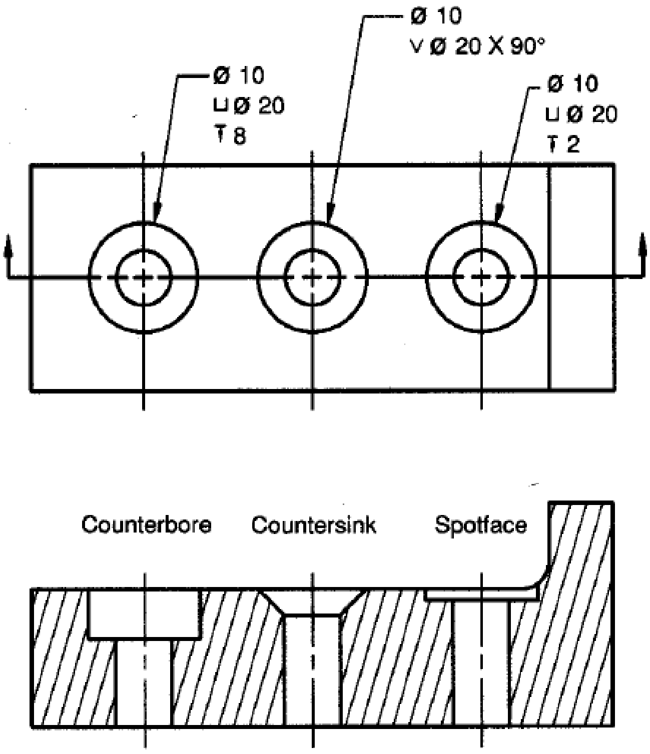


Figure: Metric Notes for Counterbored, Countersunk and Spotfaced Holes.

Because of this, the thru hole requires no specification of depth in the note. The word "THRU" should not be included with the note. If no depth is specified, a hole is by default a thru feature. This is demonstrated in the notes for the countersunk and counterbored holes shown in the Figures.

A blind hole is machined to a specified depth. This depth specification must be included in the note for a blind hole. The depth value refers to the cylindrical (useable) portion of the hole (see the next Figure). The tip angle in not included in the value of hole depth. When multiple occurrences of the same hole specification exist in a single component, it is not necessary to write a callout to each hole in the pattern. Rather, the preferred procedure is to write the note to one hole, and then include within that note a reference to the total number of identical features in the pattern. The proper form for these notes is given below and in the figure at right.
4 x φ.375

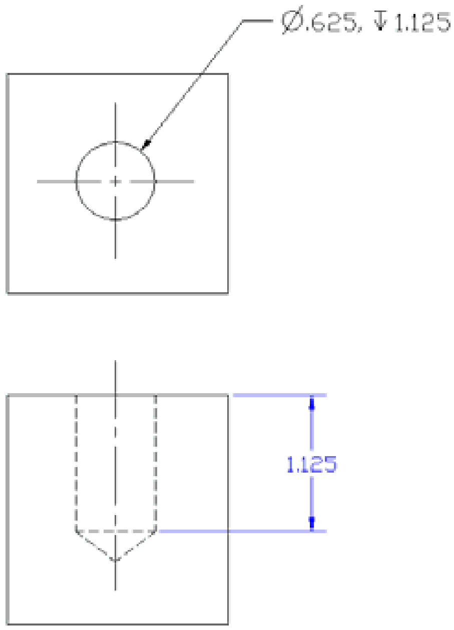


Figure: Hole Depth

Writing notes for threaded holes:

The note for a threaded hole is a specification of all information required for the creation of the hole. This includes; 1) the diameter (and depth if blind) of the pilot hole drilled prior to thread creation. 2) the specification of the internal threads for the hole. Again a depth is given if the hole is blind.

The creation of the internal threads is a metal cutting process referred to as "tapping".

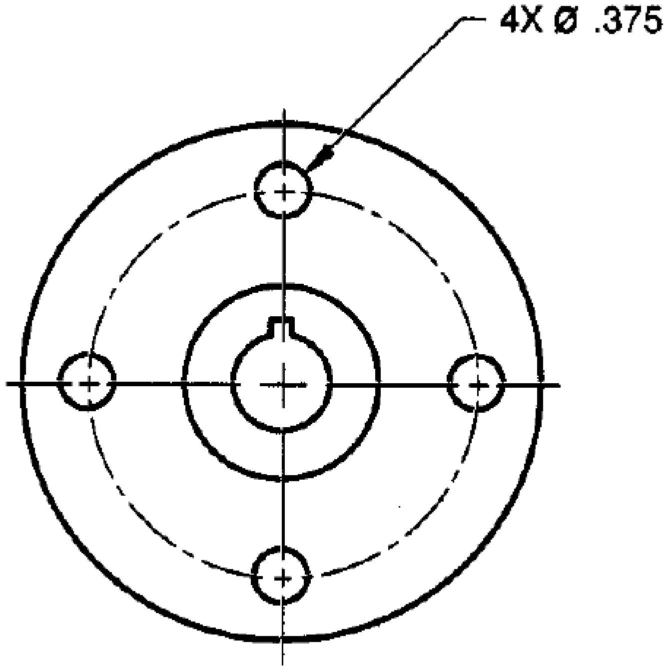


Figure: Multiple Occurrences

It should be apparent that in order to cut metal, the diameter of the pilot hole must be smaller than the major diameter of the threads. This difference in diameters is very important. If the pilot hole diameter is too small, too much material will have to be cut and the thread cutting tool (tap), being very hard (and therefore brittle) will break. If the pilot hole diameter is too large, the thread height will be too small and load carrying capability of threads will be compromised. In practice, the diameter of the pilot hole will set the minor diameter of the internal threads. Typically the thread height for internal threads is approximately 75% of the mating external threads (it may be as low as 50% for materials such as steel). This means a gap will exist between the crest of the external thread and the root of the internal. For this reason, threads may not be considered a seal in and of themselves.

The diameter of the pilot hole is specific for each thread series and form. This unique diameter is determined by referencing the thread series and form within a standard table. Typically this value is referred to in the table as the "tap drill diameter". (although in the table below it is given as "Drill Size") The following table also provides the values of *Threads per Inch* for specific nominal diameters and thread series. Notice that in the table shown above, the tap drill diameter is given in fractions, letters, and numbers. These are all drill sizes, just designated in different ways. When including these diameters in the annotation, use the following.

Diameter from table a fraction:	write as exact decimal equivalent or fraction
Diameter from table a letter:	write letter and give decimal equivalent* as reference (in parentheses)
Diameter from table a number:	write number and give decimal equivalent* as reference
*** these values may be obtained from Number and Letter Drill Size decimal equivalence tables**	

The note for the threaded hole is then written in order of operation. That is, the specification of the pilot hole, then the specification of the threads being cut, and depth (if required)

Table: Tap Chart - UNC/UNF Threads

Tap size	NF/NC UNF/UNC	Threads per inch	Basic major dia (inches)	Basic effective dia (inches)	Basic minor dia of int threads (inches)	Basic minor dia of ext. threads (inches)	Drill size
1/4-20	UNC	20	.2500	.2175	.1887	.1959	#7
1/4-28	UNF	28	.2500	.2268	.2062	.2113	#3
5/16-18	UNC	18	.3125	.2764	.2443	.2524	F
5/16-24	UNF	24	.3125	.2854	.2614	.2674	I
3/8-16	UNC	16	.3750	.3344	.2983 .	3073	5/16**.**
3/8-24	UNF	24	.3750	.3479	.3239	.3299	Q
7/16-14	UNC	14	.4375	.3911	.3499	.3602	U
7/16-20	UNF	20	.4375	.4050	.3762	.3834	25/64
1/2-13	UNC	13	.5000	.4500	.4056	.4167	27/64
1/2-20	UNF	20 .	5000	.4675	.4387	.4459	29/64
9/16-12	UNC	12	.5625	.5084	.4603	.4723	31/64
9/16-18	UNF	18	.5625	.5264	.4943	.5024	33/64
5/8-11	UNC	11	.6250	.5660	.5135	.5266	17/32
5/8-18	UNF	18	.6250	.5869	.5568	.5649	37/64
3/4-10	UNC	10	.7500	.6650	.6273	.6417	21/32
3/4-16	UNF	16	.7500	.7094	.6733	.6823	11/16
7/8-9	UNC	9	.8750	.8028	.7387	.7547	49/64
7/8-14	UNF	14	.8750	.8286	.7874	.7977	13/16
1-8	UNC	8	1.000	.9188	.8466	.8647	7/8
1-14	UNF	14	1.000	.9459	.8978	.9098	15/16

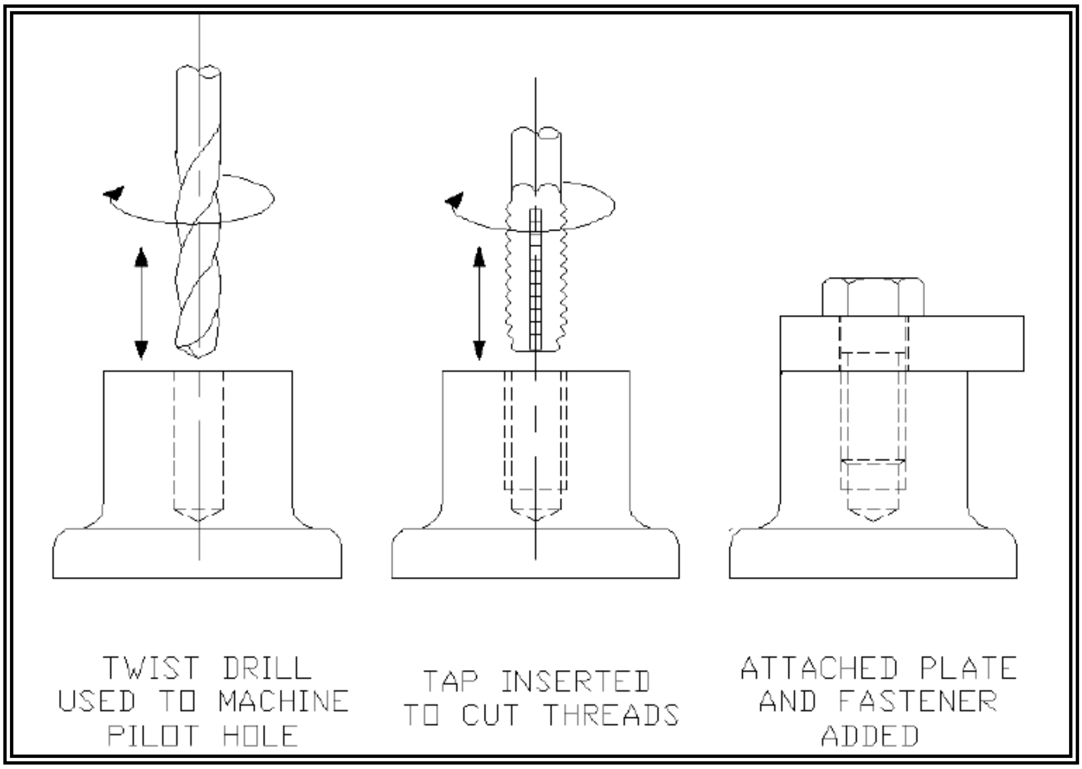


Figure: Machining a Threaded Hole

Examples of notes for threaded holes.

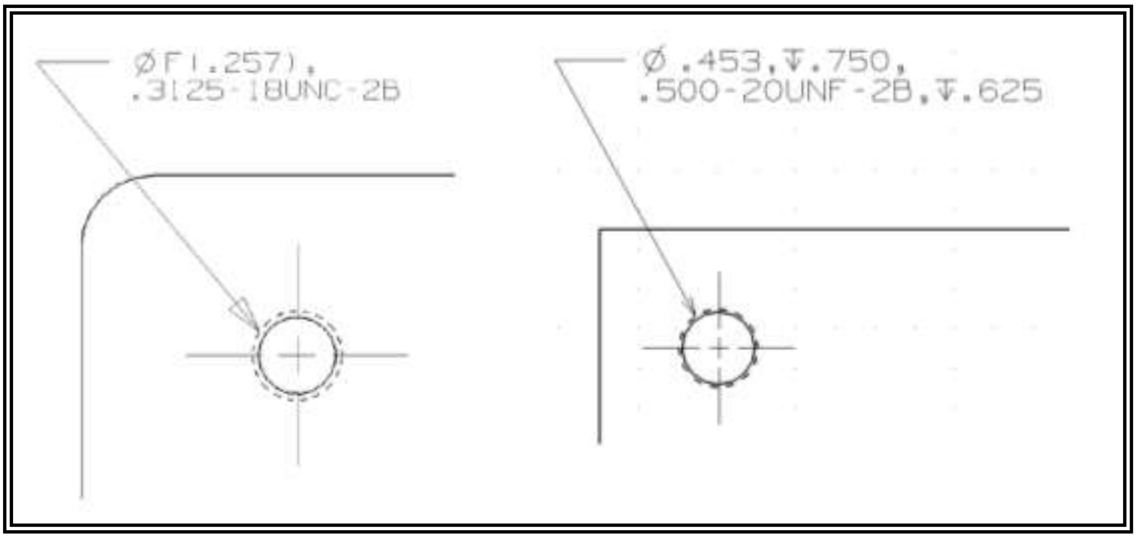


Figure: Examples of notes for threaded holes.

Threaded Mechanical Fasteners

In order to fully understand engineering prints and to provide adequate information when ordering components, one should be able to both create and read complete mechanical fastener specifications. This will give you the ability to write accurate specification of desired fastener and to associate a given specification with the respective fastener.

The specification of a fastener includes the following:

A Complete Thread Specification

Head type

Fastener type

Fastener length

It also may include a specification of material and grade (strength).

Examples of fastener specification for the various fastener types are given later in this course notes.

There exist many different head types for mechanical fasteners. Some are very specialized such as castellated and tamper proof heads. We will only consider six basic head types. These six basic types are listed below along with the standard abbreviation for each.

Hexagonal head (HEX HD)

Fillister head (FIL HD

Flat head (FLAT HD)

Oval head (OVAL HD)

Round head (RND HD)

Hexagonal socket head (SOC HD)

Note: The fillister, flat, oval and round head types are commonly available with slot or Phillips drive. Other drive types (such as hex socket) are also available, but less common

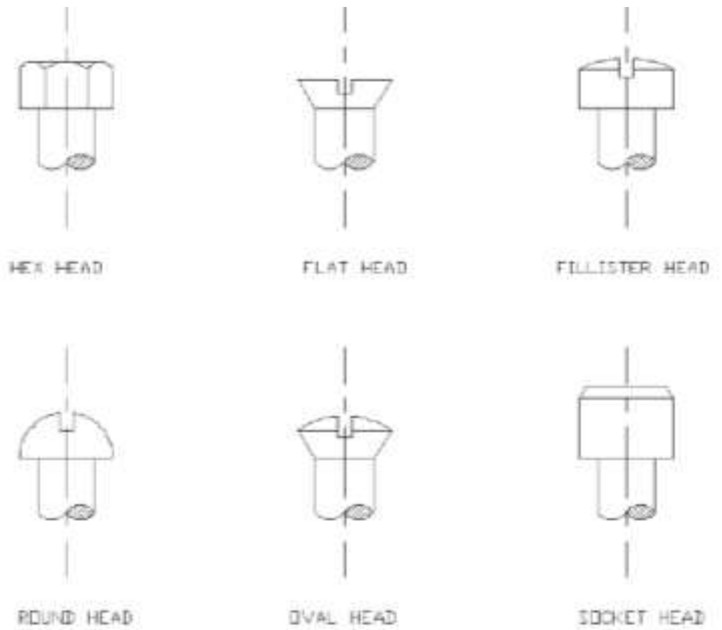


Figure: Common Head Types

Mechanical Fasteners:

There are three basic types of mechanical fastener. They are the
Cap Screw (CAP SCR)
Machine Screw (MACH SCR) and
The Set Screw (SET SCR).

Cap screws and machine screws are very similar. Both are available with the same type of head. They are both used in conjunction with internally threaded holes for the purpose of clamping components together. There are however, difference between cap and machine screws.

Clamping Force:

When a cap or machine screw is used to attach to components to one another, the fastener is inserted through a clearance hole in one component and onto a threaded hole in another. An alternative assembly would be to pass the fastener through two clearance holes and use a nut for clamping. Clamping force is applied through contact between the bottom face of head and the contact between the internal and external threads. When these methods are used, the fastener is inserted into the internally threaded component (either the threaded hole or the nut) and advanced by rotating the fastener. When the head of the fastener make contact with surface of the component being attached, the head can advance no further. However, some additional rotation of the fastener can be made, usually by means of some fashion or tool (a wrench for example). Since the threads will advance during this rotation but the head cannot a tensile load is generated in the shank of the fastener. This tensile load is proportional to the force used to rotate the fastener. The rotational force is referred to as "seating torque" and the tensile force is referred to as "pre-load".

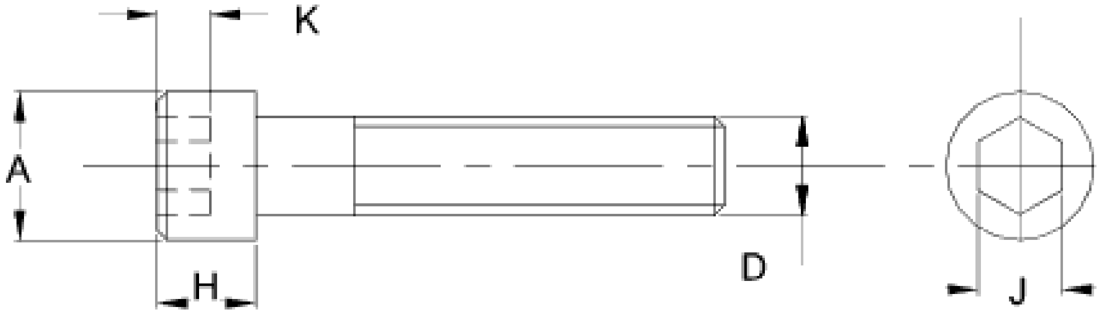
Cap Screws (CAP SCR)

Cap screws tend toward larger diameters. The threaded end of a cap screw is chamfered. The minimum thread length is a function fastener nominal diameter. For most cap screws, the minimum length of thread equals 2 * DIA + 0.25. For socket head cap screws, the minimum thread length equals 2 * DIA + 0.50. A cap screw specified with a nut is referred to as a bolt.

Metric Cap Screws Tables

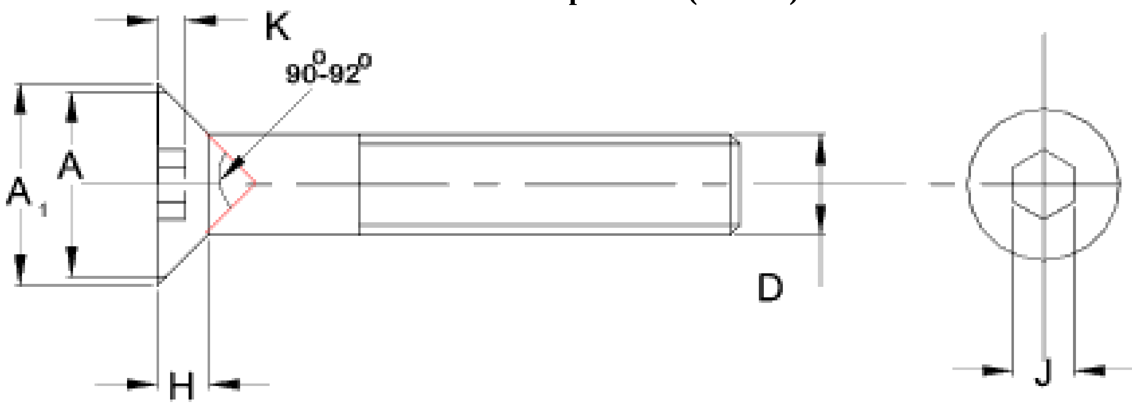
Notes that all linear dimensions in millimeters The dimensions are generally in accordance with BS EN ISO 4762 BS 3643- 2 & BS 4168

Table: Socket Head Cap Screws (metric)



Nominal Size	Thread Pitch	Hex Socket Size [J]	Body diameter [D] and Head height [H]		Head Diameter [A]		Soc length [K].
			Max	Min	Max	Min	
M3	0.5	2.50	3.00	2.86	5.50	5.20	1.3
M4	0.70	3.00	4.00	3.82	7.00	6.64	2.00
M5	0.8	4.00	5.00	4.82	8.50	8.14	2.70
M6	1.0	5.00	6.00	5.82	10.00	9.64	3.30
M8	1.25	6.00	8.00	7.78	13.00	12.57	4.3
M10	1.5	8.00	10.00	9.78	16.00	15.57	5.50
M12	1.75	10.00	12.00	11.73	18.00	17.57	6.60
M16	2.0	14.00	16.00	15.73	24.00	23.48	8.80
M20	2.5	17.00	20.00	19.67	30.00	29.48	10.70
M24	3.0	19.00	24.00	23.67	36.00	35.38	12.90

Table Flat Head Cap Screws (Metric)



Nominal Size [D]	Thread Pitch	Hex Socket Size [J]	Max Cone Dia [A1]	Head Dia		Head Height [H]	Soc. Length [K]
				[A]_max	[A]_Min		
M3	0.5	2,0	6,72	6,00	5,82	1,86	1,05
M4	0.70	2,5	8,96	8,00	7,78	2,48	1,49
M5	0.8	3,0	11,2	10,00	9,78	3,1	1,86
M6	1.0	4,0	13,44	12,00	11,75	3,72	2,16
M8	1.25	5,0	17,92	16,00	15,73	4,96	2,85
M10	1.5	6,0	22,4	20,00	19,67	6,2	3,60
M12	1.75	8,0	26,88	24,00	23,67	7,44	4,35
M16	2.0	10,0	33,6	32,00	29,67	8,8	4,89
M20	2.5	10,0	40,32	40,00	35,61	10,16	5,49

Machine Screws (MACH SCR)
Machine screws are only available in smaller diameters. The threaded end of the fastener not chamfered but rather simply sheared. The minimum thread length is a function of fastener length as follows:
if fastener length > 2, then min. thread length = 1.75
if fastener length < 2, then min. thread length = fastener length
Examples of Cap and Machine Screw Fastener Descriptions
The following example is the specification for a 1.50 long cap screw with a hexagonal head and using 7/16 nominal diameter Unified fine threads of a standard fit.

1.50 X .4375 – 20UNF –2A HEX HD, CAP SCR

Set Screws (SET SCR)
The function of set screws is to restrict or control motion.. They are commonly used in conjunction with collars, pulleys, or gears on shafts. With the exception of the antiquated square head, set screws are headless fasteners and therefore threaded for their entire length. Lacking heads, set screws are categorized by drive type (similar to head type) and point style. Most set screws use Class 3 fit threads. This is to provide resistance to the set screw "backing out" of its threaded hole during service. In addition, set screws have a specified point type. The point is used to provide various amounts of holding power when used. Holding power concerns will be discussed below. The available point types for set screws are the Cone, Cup, Flat, Oval, and Dog (full or half) points. Profiles of these point type are shown in the following figure.

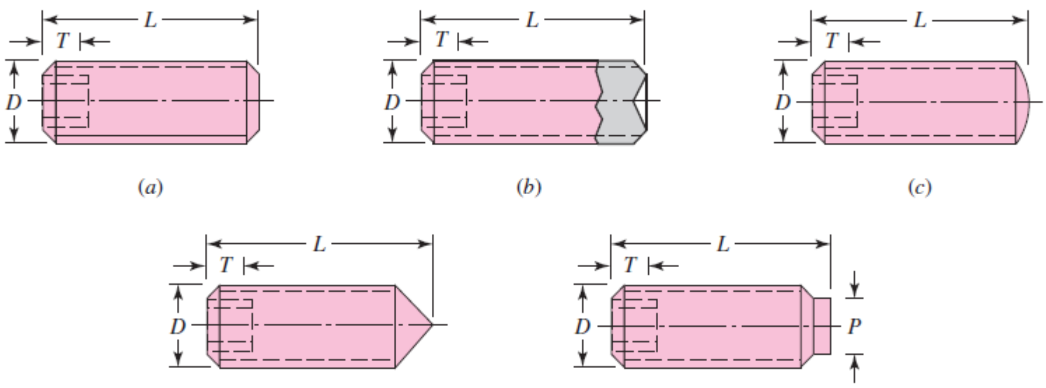


Figure Drawings of different types of setscrews or grub screws. Socket setscrews: (*a*) flat point; (*b*) cup point; (*c*) oval point; (*d*) cone point; (*e*) half-dog point.(Repeated)

Set Screw Holding Power:
The following table lists values of the seating torque and the corresponding holding power for inch-series setscrews. The values listed apply to both axial holding power, for resisting thrust, and the tangential holding power, for resisting torsion. Typical factors of safety are 1.5 to 2.0 for static loads and 4 to 8 for various dynamic loads. Setscrews should have a length of about half of the shaft diameter. Note that this practice also provides a rough rule for the radial thickness of a hub or collar.

Table : Typical Holding Power (Force) for Socket Setscrews* *Source:* Unbrako Division, SPS Technologies, Jenkintown, Pa.

Size, . in	Seating Torque, Lbf. in	Holding Power, lbf	Size, . in	Seating Torque, Lbf. in	Holding Power, lbf
#0	1.0	50	5/16	165	1500
#1	1.8	65	3/8	290	2000
#2	1.8	85	7/16	430	2500
#3	5	120	1/2	620	3000
#4	5	160	9/16	620	3500
#5	10	200	5/8	1325	4000
#6	10	250	3/4	2400	5000
#8	20	385	7/8	5200	6000
#10	36	540	1	7200	7000
1/4	87	1000			

*Based on alloy-steel screw against steel shaft, class 3A coarse or fine threads in class 2B holes, and cup-point socket setscrews

In many applications, set screws are used to prevent the rotational and axial movement of parts such as collars, couplings, and pulley sheaves mounted to shafts. Failure of the set screw in these cases is relative motion of .01 inch between components. An important consideration in setscrew selection is the holding power provided by the contact between the setscrew point and attachment surface (typically a cylindrical shaft). Holding power is generally specified as the tangential force in pounds. Axial holding power is assumed to be equal to the torsional holding power. Some additional resistance to rotation is contributed by penetration of the set screw point into the attachment surface. In cases where point penetration is desired, the set screw should have a material hardness at least 10 points higher on the Rockwell scale than that of the attachment material. Cup-point set screws cut into the shaft material. Cone-point setscrews also penetrate the attachment surface and may be used with a spotting hole to enhance this penetration. Oval-point and flat-point setscrews do not penetrate the surface and hence have less holding power.

Set screw selection often begins with the common axiom stating that set screw diameter should be equal to approximately one-half shaft diameter. This rule of thumb often gives satisfactory results, but its usefulness may be limited. Manufacturers' data or data supplied by standard machine design texts will give more reliable results.

Seating torque: Torsional holding power is almost directly proportional to the seating torque of cup, flat, and oval-point setscrews.

Point style: Setscrew point penetration contributes as much as 15% to the total holding power. When the cone-point setscrew is used, it requires the greatest installation torque because of its deeper penetration. Oval point, which has the smallest contact area, yields the smallest increase in holding power.

Relative hardness: Hardness becomes a significant factor when the difference between setscrew point and shafting is less than 10 Rockwell C scale points. Lack of point penetration reduces holding power.

Flatted shafting: About 6% more torsional holding power can be expected when a screw seats on a flat surface. Flatting, however, does little to prevent the 0.01-in. relative movement usually considered as a criterion of failure. Axial holding power is the same.

Length of thread engagement: The length of thread engagement does not have a noticeable effect on axial and torsional holding power, provided there is sufficient engagement to prevent thread stripping during tightening. In general, the minimum recommended length of engagement is 1 to 1.5 times the major diameter of the setscrew for threading in brass, cast iron, and aluminum; and 0.75 to 1 times the diameter for use in steel and other materials of comparable hardness. Be aware that the lengths of
engagement specified are for full threads engaged, not overall screw length.

Thread type: A negligible difference exists in the performance of coarse and fine threads of the same class of fit. Most set screws are class 3A fit.

Drive type: Most set screws use socket (either hex or fluted) drive or a slot drive. The type of drive affects the seating torque that can be attained because it determines how much torque can be transmitted to the screw. Less torque can be transmitted through a slot drive than a socket drive. Therefore, holding power of the slotted screw is about 45% less.

Number of setscrews: Two setscrews give more holding power than one, but not necessarily twice as much. Holding power is approximately doubled when the second screw is installed in an axial line with the first but is only about 30% greater when the screws are diametrically opposed. Where design dictates that the two screws be installed on the same circumferential line, displacement of 60° is recommended as the best compromise between maximum holding power and minimum metal between tapped holes. This displacement gives 1.75 times the holding power of one screw.

Torque force: The compressive force developed at the point depends on lubrication, finish, and material.

Setscrews and keyways: When a setscrew is used in combination with a key, the screw diameter should be equal to the width of the key. In this combination, the setscrew holds the parts in an axial direction only. The key, keyseat and keyway assembly carries the torsional load on the parts. The key should be tight fitting so that no motion is transmitted to the screw. Under high reversing or alternating loads, a poorly fitted key will cause the screw to back out and lose its clamping force.

Examples of Set Screw Fastener Descriptions

The following example is the specification for a 1.00 long set screw with a hexagonal socket drive, a cup point, a 1/4 nominal diameter, Unified fine threads and a class 3 fit.

1.00 X .250 – 28UNF – 3A SOC HD, CUP PT, SET SCR

Eccentric Loaded bolted Joint

When the line of action of the load does not pass through the centroid of the bolts system and thus all bolts are not equally loaded, then the joint is said to be an eccentric loaded bolted joint, as shown in the following Figure. The eccentric loading results in secondary shear caused by the tendency of force to twist the joint about the centre of gravity in addition to direct shear or primary shear. Riveted and bolted joints loaded in shear are treated exactly alike in design and analysis. **(will be treated later)**

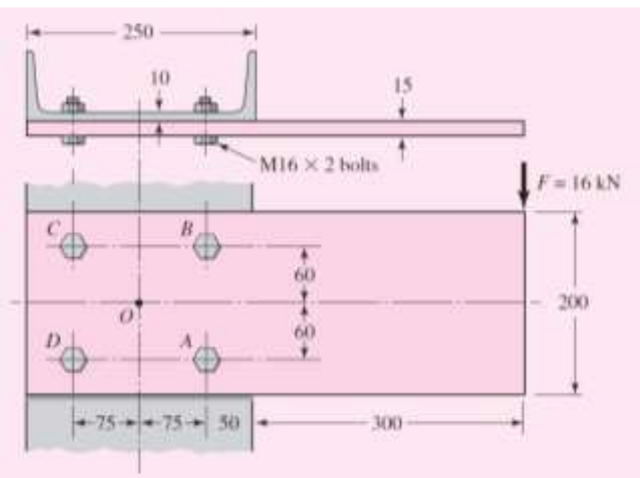


Figure Eccentric Loaded bolted Joint

(Dimensions in millimeters).

Power Screw

Power screws and ball screws are designed to convert rotary motion to linear motion and to exert the necessary force to move a machine element along a desired path. Power screws operate on the classic principle of the screw thread and its mating nut. If the screw is supported in bearings and rotated while the nut is restrained from rotating, the nut will translate along the screw. If the nut is made an integral part of a machine, for example, the tool holder for a lathe, the screw will drive the tool holder along the bed of the machine to take a cut. Conversely, if the nut is supported while it is rotating, the screw can be made to translate. The screw jack uses this approach.

A ball screw is similar in function to a power screw, but the configuration is different. The nut contains many small, spherical balls that make rolling contact with the threads of the screw, giving low friction and high efficiencies when compared with power screws. Modern machine tools, automation equipment, vehicle steering systems, and actuators on aircraft use ball screws for high precision, fast response, and smooth operation. Visit a machine shop where there are metal-cutting machine tools. Look for examples of power screws that convert rotary motion to linear motion. They are likely to be on manual lathes moving the tool holder. Or look at the table drive for a milling machine. Inspect the form of the threads of the power screw. Are they of a form similar to that of a screw thread with sloped sides? Or are the sides of the threads straight? Compare the screw threads with those shown in the following figure for square, Acme, and buttress forms.

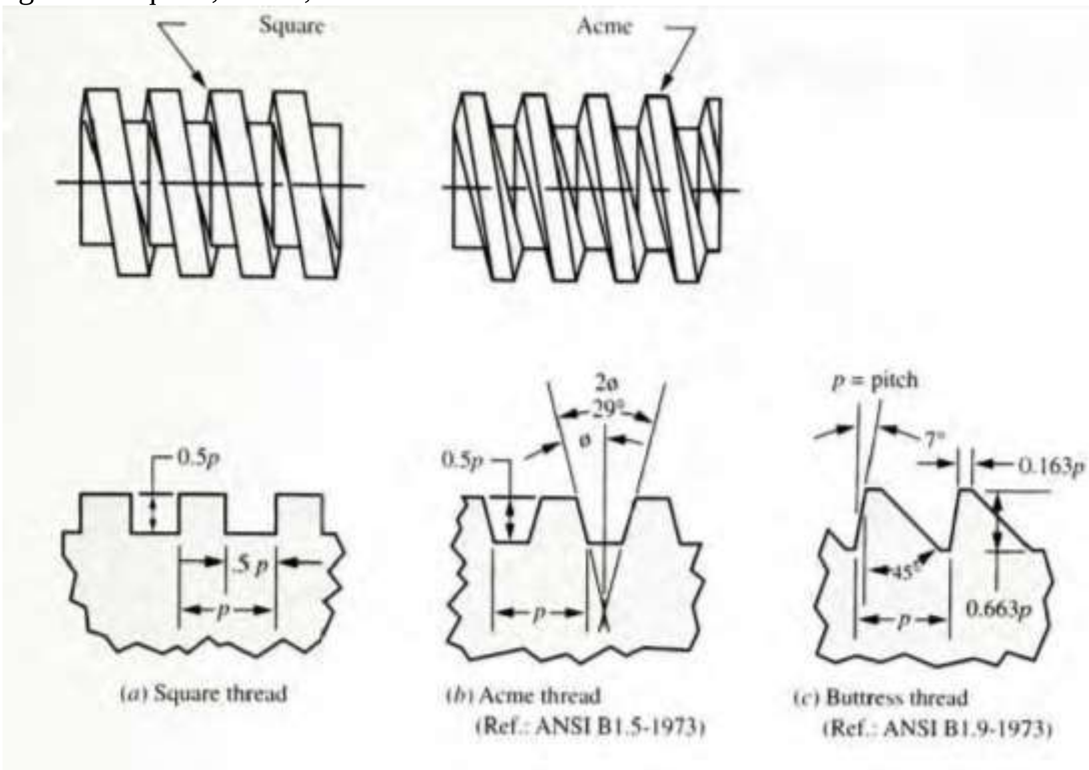


Figure : Forms Of Power Screw Threads [(a) Square (b), Acme (ANSI Standard B 1.5-1973); (c) Buttress (ANSI Standard B 1.9-1973)]

While in the shop, do you see any type of material-testing equipment or a device called an *arbor press* that exerts large axial forces? Such machines often employ square thread power screws to produce the axial force and motion from rotational input, through either a hand crank or an electric motor drive. If they are not in the machine shop, look for them in the metallurgy lab or another room where materials testing is done. Now look further in the machine shop. Are there machines that use digital readouts to indicate position of the table or the tool? Are there computer numerical control machine tools? Any of these types of machines should have ball screws rather than the traditional power screws because ball screws require significantly less power and torque to drive them against a given load. They can also be moved faster and positioned more accurately than power screws. You may or may not be able to see the recirculating balls in the nut of the power screw. But you should be able to see the different shaped threads looking like grooves with circular bottoms in which the spherical balls roll. Have you .seen such power screws or ball screws anywhere outside a machine shop? Some garage door openers employ a screw drive, but others use chain drives. Perhaps your home has a .screw jack or a scissors jack for raising the car to change a tire. Both use power screws. Have you ever sat in a seat on an airplane where you can see the mechanisms that actuate the flaps on the rear edge of the wings? Try it .sometime, and observe the actuators during takeoff or landing. It is likely that you will see a ball screw in action. This chapter will help you learn the methods of analyzing the performance of power screws and ball screws and to specify the proper size for a given application. **[Ref. Mott, 2004]**

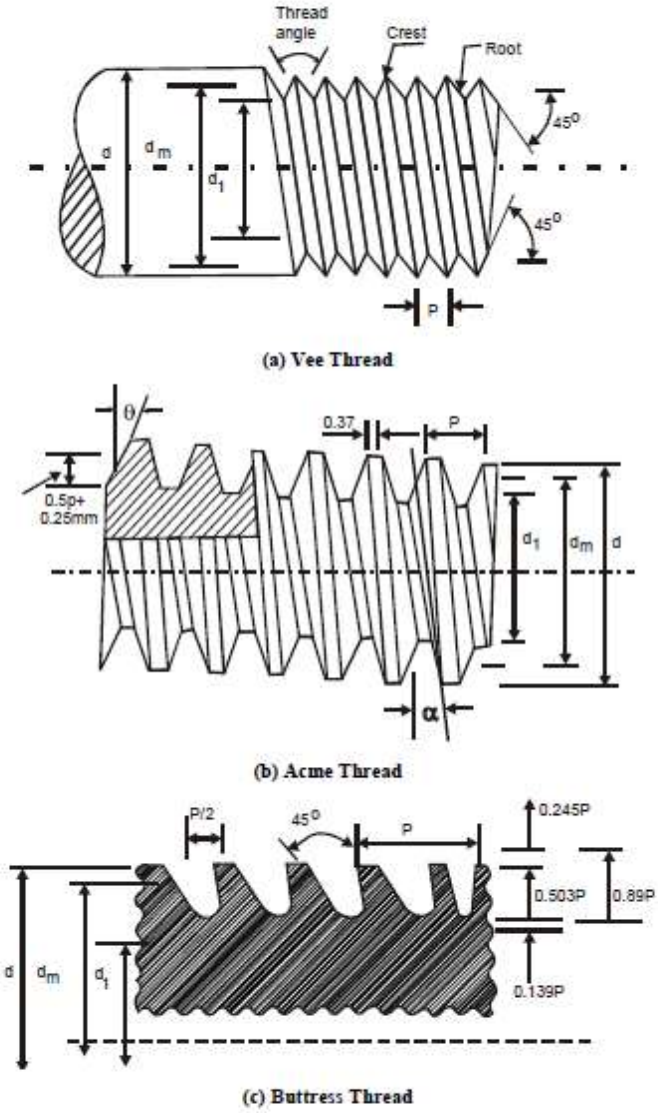


Figure : Types Of Power Screw Threads

Power Screw Dimensions: The above Figure shows three types of power screw threads: the square thread, the Acme thread, and the buttress thread. Of these, the square and buttress threads are the most efficient. That is, they require the least torque to move a given load along the screw. However, the Acme thread is not greatly less efficient, and it is easier to machine. The buttress thread is desirable when force is to be transmitted in only one direction. The following Tables gives the preferred combinations of basic major diameter, *D.* and number of threads per inch, *n.* for Acme screw threads. The pitch, *p.* is the distance from a point on one thread to the corresponding point on the adjacent thread, and $p = 1/n$. Other pertinent dimensions listed in the Table include the minimum minor diameter and the minimum pitch diameter of a screw with an external thread. When you are performing stress analyses on the screw, the safest approach is to compute the area corresponding to the minor diameter for tensile or compressive stresses. However, a more accurate stress computation results from using the *tensile stress area* (listed in the Table).

TABLE Preferred Acme screw threads

Nominal major diameter, D	**Threads per in,**	**Pitch. p = 1/n**	**Minimum minor diameter, D_r**	**Minimum pitch diameter, D_p**	**Tensile stress area, A_t**	**Shear stress area, A_s**
(in)	**n**	**(in)**	**(in)**	**(in)**	**(in^2)**	**(in^2)**
1/4	16	0.0625	0.1618	0.2043	0.026 32	0.3355
5/16	14	0.0714	0.2140	0.2614	0.044 38	0.4344
3/8	12	0.0833	0.2632	0.3161	0.065 89	0.5276
7/16	12	0.0833	0.3253	0.3783	0.097 20	0.6396
1/2	10	0.1000	0.3594	0.4306	0.1225	0.7278
5/8	8	0.1250	0.4570	0.5408	0.1955	0.9180
3/4	6	0.1667	0.5371	0.6424	0.2732	1.084
7/8	6	0.1667	0.6615	0.7663	0.4003	1.313
1	5	0.2000	0.7509	0.8726	0.5175	1.493
1 1/8	5	0.2000	0.8753	0.9967	0.6881	1.722
1 1/4	5	0.2000	0.9998	1.1210	0.8831	1.952
1 3/8	4	0.2500	1.0719	1.2188	1.030	2.110
1 1/2	4	0.2500	1.1965	1.3429	1.266	2.341
1 *3/4*	4	0.2500	1.4456	1.5916	1.811	2.803
2	4	0.2500	1.6948	1.8402	2.454	3.262
2 1/4	3	0.3333	1.8572	2.0450	2.982	3.610
2 1/2	3	0.3333	2.1065	2.2939	3.802	4.075
2 3/4	3	0.3333	2.3558	2.5427	4.711	4.538
3	2	0.5000	2.4326	2.7044	5.181	4.757
3 1/2	2	0.5000	2.9314	3.2026	7.388	5.700
4	2	0.5000	3.4302	3.7008	9.985	6.640
4 1/2	2	0.5000	3.9291	4.1991	12.972	7.577
5	2	0.5000	4.4281	4.6973	16.351	8.511

Alternative Thread Forms for Power Screws

Modifications are frequently made to both Acme and square threads. For instance, the square thread is sometimes modified by cutting the space between the teeth so as to have an included thread angle of 10 to 15°. This is not difficult, since these threads are usually cut with a single-point tool anyhow; the modification retains most of the high efficiency inherent in square threads and makes the cutting simpler. While the standard Acme thread is probably the most widely used, others are available. The *stub Acme* thread has a similar form with a 29° angle between the sides, the depth of the thread is shorter, providing a stronger, more rigid thread. Metric power screws are typically made according to the ISO trapezoidal form that has a 30° included angle. The relatively low efficiency of standard single-thread Acme screws (approximately

30% or less) can be a strong disadvantage. Higher efficiencies can be achieved using high lead, multiple thread designs. The higher lead angle produces efficiencies in the 30% to 70% range. It should be understood that some mechanical advantage is lost so that higher torques are required to move a particular load as compared with single thread screws.

Table: Basic Dimensions of Square Thread, (mm)

Pitch, p	**5**							
Core Dia. d_1	17		19		24		23	
Major Dia. d	22		24		26		28	
Pitch, p	**6**							
Core dia. d_1	24		26		28		30	
Major dia. d	30		32		34		36	
Pitch, p	**7**							
Core dia. d_1	31		33		35		37	
Major dia. d	38		40		43		44	
Pitch, p	**8**							
Core Dia. d_1	38		40		42		44	
Major Dia. d	46		48		50		56	
Pitch, p	**9**							
Core dia. d_1	46		49		51		53	
Major dia. d	55		58		60		62	
Pitch, p	**10**							
Core Dia. d_1	55	58	60	62	65	68	70	72
Major Dia. d	65	68	70	72	75	78	80	82

The Forces and torques analysis of Power Screws

Familiar applications include the lead screws of lathes, and the screws for vises, presses, Universal testing machine, and jacks. In order to design these applications forces and torques must be calculated. The next figure shows a free body diagram of one thread of a square-threaded power screw moves on its nut. The thread having a mean diameter d_m, a pitch p, a lead angle λ, and a helix angle ψ is loaded by the axial compressive force F. We wish to find an expression for the torque required to raise this load, and another expression for the torque required to lower the load.

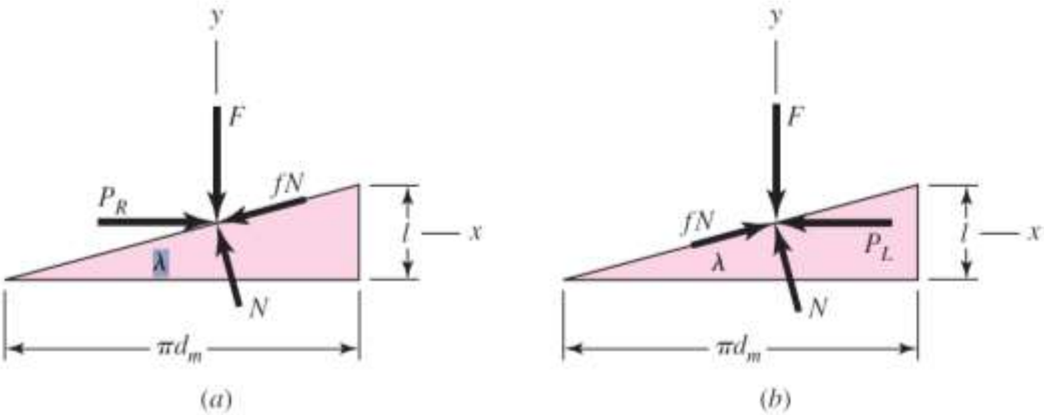


Figure : Force diagrams: (*a*) lifting the load; (*b*) lowering the load.

First, imagine that a single thread of the screw is unrolled or developed for exactly a single turn. Then one edge of the thread will form the hypotenuse of a right triangle whose base is the circumference of the mean thread diameter circle and whose height is the lead. The angle $\boldsymbol{\lambda}$ is the lead angle of the thread. We represent the summation of all the axial forces acting upon the normal thread area by $\boldsymbol{F}$.
To raise the load, a force $\boldsymbol{P_R}$ acts to the right (part *a* of the figure), and to lower the load, $\boldsymbol{P_L}$ acts to the left (part *b*). The friction force is the product of the coefficient of friction $\boldsymbol{f}$ with the normal force $\boldsymbol{N}$, and acts to oppose the motion. The system is in equilibrium under the action of these forces, and hence, for raising the load, we have

$$\left.\begin{aligned}\sum F_x &= P_R - N\sin\lambda - f\,N\cos\lambda = 0\\ \sum F_y &= -F - f\,N\sin\lambda + N\cos\lambda = 0\end{aligned}\right\}\ldots\text{(a)}$$

In a similar manner, for lowering the load, we have

$$\left.\begin{aligned}\sum F_x &= -P_L - N\sin\lambda + f\,N\cos\lambda = 0\\ \sum F_y &= -F + f\,N\sin\lambda + N\cos\lambda = 0\end{aligned}\right\}\ldots\text{(b)}$$

Since we are not interested in the normal force $\boldsymbol{N}$, we eliminate it from each of these sets of equations and solve the result for $\boldsymbol{P}$. For raising the load, this gives

$$P_R = \frac{F\,(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda} \qquad \ldots\text{(c)}$$

and for lowering the load,

$$P_L = \frac{F\,(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda} \qquad \ldots\text{(d)}$$

Next, divide the numerator and the denominator of these equations by cos λ and use the relation tan $\boldsymbol{\lambda = l/\pi dm}$ (see figure). We then have, respectively,

$$P_R = \frac{F[\,(l/\pi d_m)+f]}{1-(fl/\pi d_m)} \qquad \ldots\text{(e)}$$

$$P_L = \frac{F[\,f-(l/\pi d_m)]}{1+(fl/\pi d_m)} \qquad \ldots\text{(f)}$$

Finally, noting that the torque is the product of the force P and the mean radius $d_m/2$, for raising the load we can write

$$T_R = \frac{Fd_m}{2}\left(\frac{l+\pi f d_m}{\pi d_m - fl}\right) \qquad \ldots\text{(g)}$$

where T_R is the torque required for two purposes: to overcome thread friction and to raise the load.
The torque required to lower the load, from Eq. (f), is found to be

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + fl}\right) \qquad \ldots\text{(h)}$$

This is the torque required to overcome a part of the friction in lowering the load. It may turn out, in specific instances where the lead is large or the friction is low, that the load will lower itself by causing the screw to spin without any external effort. In such cases, the torque T_L from Eq. (h) will be negative or zero. When a positive torque is obtained from this equation, the screw is said to be *self-locking*. Thus the condition for self-locking is

$$\pi f d_m > l \qquad \ldots\text{(i)}$$

Now divide both sides of this inequality by πd_m. Recognizing that $l/\pi d_m = \tan\lambda$, we get

$$f > \tan\lambda \qquad \text{…(j)}$$

This relation states that self-locking is obtained whenever the coefficient of thread friction is equal to or greater than the tangent of the thread lead angle.

An expression for efficiency is also useful in the evaluation of power screws. If we let $f = 0$ in Eq. (g), we obtain

$$T_0 = Fl\,/\,2\pi \qquad \text{...(k)}$$

which, since thread friction has been eliminated, is the torque required only to raise the load. The efficiency is therefore

$$\mu = \frac{T\ without\ friction}{T\ with\ friction} = \frac{T_0}{T_R} = \frac{F\,l}{2\,\pi\,T_R} = \frac{\tan\lambda}{\tan(\lambda+\psi)} \qquad \text{…(l)}$$

The preceding equations have been developed for square threads where the normal thread loads are parallel to the axis of the screw. In the case of Acme or other threads, the normal thread load is inclined to the axis because of the thread angle 2α and the lead angle λ. Since lead angles are small, this inclination can be neglected and only the effect of the thread angle (see the next figure) considered. The effect of the angle α is to increase the frictional force by the wedging action of the threads.

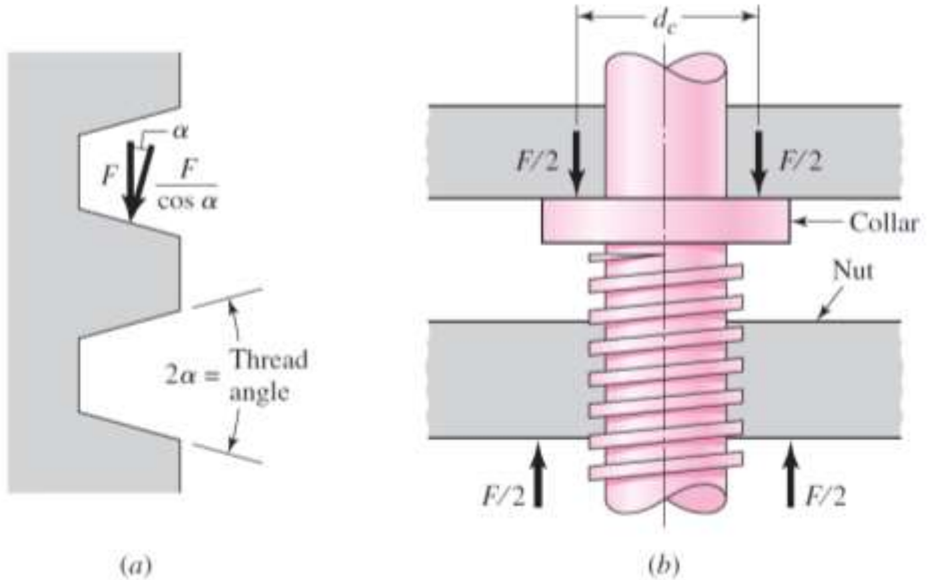


Figure : (*a*) Normal thread force is increased because of angle *α*; (*b*) thrust collar has frictional diameter *dc*.

Therefore the frictional terms in Eq. (g) must be divided by cos *α*. For raising the load, or for tightening a screw or bolt, this yields

$$T_R = \frac{Fd_m}{2}\left(\frac{l+\pi f d_m \sec\alpha}{\pi d_m - fl\sec\alpha}\right) \qquad \text{…(m)}$$

In using Eq. (m), remember that it is an approximation because the effect of the lead angle has been neglected.

For power screws, the Acme thread is not as efficient as the square thread, because of the additional friction due to the wedging action, but it is often preferred because it is easier to machine and permits the use of a split nut, which can be adjusted to take up for wear.

Usually a third component of torque must be applied in power-screw applications. When the screw is loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members in order to carry the axial component. Part (b) of the above figure shows a typical thrust collar in which the load is assumed to be concentrated at the mean collar diameter *dc*. If f_c is the coefficient of collar friction, the torque required is

$$T_c = \frac{F f_c d_c}{2} \qquad \ldots(n)$$

For large collars, the torque should probably be computed in a manner similar to that employed for disk clutches.

Example 1:

A square threaded screw is required to work against an axial force of 6.0 kN and has following dimensions. Major diameter d = 32 mm; pitch p = 4 mm with single start, f = 0.08. Axial force rotates with the screw.

Calculate :

(a) Torque required when screw moves against the load.

(b) Torque required when screw moves in the same direction as the load.

(c) Efficiency of the screw.

Solution

Remember the relationship between p, d and d_1 which has been shown Using

d = 32 mm and p = 4 mm

d_m= 32-2= 30 mm

The angle of helix is related to the circumference of mean circle and the pitch from description above.

$$\tan\lambda = \frac{p}{\pi d_m} = \frac{4}{\pi\, 30} = 0.042$$

∴ $\lambda = 2.4^{o}$

and $\tan\alpha = f = 0.08 \rightarrow \alpha = 4.57^{o}$

Using equation (g), the torque required to move screw against load, $\mathbf{T_R}$,

$$T_R = \frac{6\times 10^3 \times 30\times 10^{-3}}{2}\left(\frac{4\times 10^{-3} + \pi\times 0.08\times 30\times 10^{-3}}{\pi\times 30\times 10^{-3} - 0.08\times 4\times 10^{-3}}\right) = 11\ N.m$$

The torque required to lower the load, from Eq. (h), is found to be

$$T_L == 3.42\ N.m$$

$$\mu = \frac{T\ without\ friction}{T\ with\ friction} = \frac{6\times 4}{2\,\pi\ \times 11} \equiv \frac{0.042}{0.12225} = 0.344 = 34.4\%$$

Example 2:

If in the Example 1, the screw has the Acme thread with thread angle 2θ = 29^{o} instead of square thread, calculate the same quantities.

Solution

There is no difference in calculation for square and the Acme thread except that in case of the Acme thread the coefficient of friction is modified and effective coefficient of friction is given by divided f by cos α

f'= 0.08/0.968 = 0.0826 and α '= 4,724 ^{o}

From Figure for the Acme thread note that

d_m =d - p/2 - 0.125 = 32- 2- 0.125 = 29.875 mm

$$\tan\lambda = \frac{p}{\pi d_m} = \frac{4}{\pi\, x\ 29.875} = 0.0426 \rightarrow \lambda = 2.44^{\circ}$$

For raising the load, or for tightening a screw or bolt, this yields

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - f l \sec\alpha}\right) = 11.265\ N.m$$

When the screw moves in the same direction as the load, the torque = 3.58 N.m

$$\mu = \frac{T\ without\ friction}{T\ with\ friction} = \frac{6 \times 4}{2\,\pi\ \times 11.265} \equiv \frac{0.0426}{0.126} = 0.338 = 33.8\%$$

Comparing the results of Examples 1 and 5 we can see that the screws have got same major diameter and pitch and for this reason their helix angles are different. Coefficients of friction are inherently different. But the torque on the screw increases by 2.41% and efficiency decreases by 1.744%.

The stress analysis of Power Screws

Nominal body stresses in power screws can be related to thread parameters as follows:

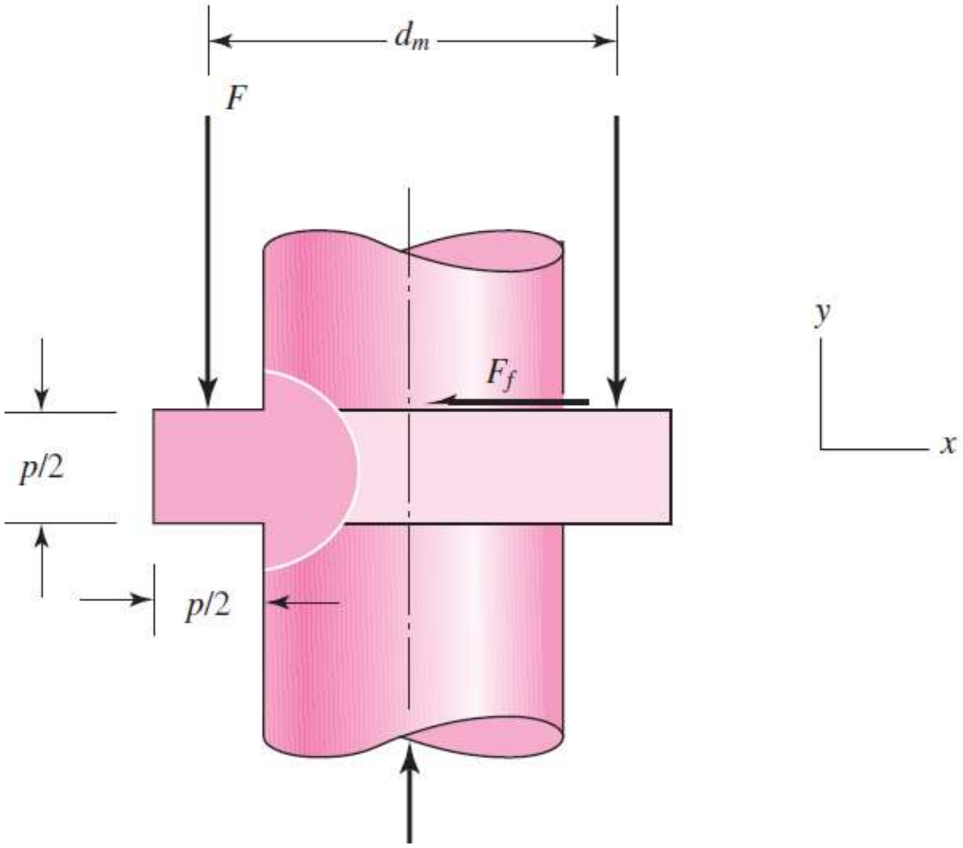


Figure: Geometry of square thread useful in finding bending and transverse shear stresses at the thread root.

- The maximum nominal shear stress τ in torsion of the screw body can be expressed as

  $\tau_p = 16T/\pi d^3$ …(o)
- The axial stress σ in the body of the screw due to load F is

  $\sigma_{tp} = F/A = 4F/\pi d_r^2$ …(p)

in the absence of column action. For a short column the J. B. Johnson buckling formula which is

$$\left(\frac{F}{A}\right)_{crit.} = S_y - \left(\frac{S_y}{2\pi}\right)^2 \frac{1}{CE} \quad \ldots(q)$$

- Nominal thread stresses in power screws can be related to thread parameters as follows: The bearing stress in the above figure, σ_{bp}, is

$$\sigma_{bp} = -\frac{F}{\pi d_m n_t\, p/2} = -\frac{2F}{\pi d_m n_t p} \qquad \ldots(r)$$

where n_t is the number of engaged threads.

- The bending stress at the root of the thread σ_b is found from

$$Z = \frac{I}{c} = \frac{(\pi d_r\, n_t)\left(\frac{p}{2}\right)^2}{6} = \frac{\pi}{24} d_r n_t p^2 \qquad M\ \frac{Fp}{4}$$

$$\sigma_{bp} = \frac{M}{Z} = \frac{Fp}{4}\frac{24}{\pi d_r n_t\, p^2} = \frac{6F}{\pi d_r n_t p} \qquad \ldots(s)$$

- The transverse shear stress τ at the center of the root of the thread due to load F is

$$\tau_p = \frac{3V}{2A} = \frac{3}{4}\frac{F}{2\,\pi d_r n_t\, p/2} = \frac{3F}{\pi d_r n_t p}\tau \qquad \ldots(t)$$

and at the top of the root it is zero. The compound at the top of the root "plane" is found by first identifying the orthogonal normal stresses and the shear stresses. From the coordinate system of the above figure, we note

$$\sigma_x = \frac{6F}{\pi d_r n_t p} \qquad \tau_{xy} = 0$$

$$\sigma_y = -\frac{4F}{\pi {d_r}^2} \qquad \tau_{yz} = \frac{16T}{\pi d_r^3}$$

$$\sigma_z = 0 \qquad \tau_{zx} = 0$$

then use the equations of two dimensional stresses(using the maximum shear theory) to find the right size of the power screw.

$$\boldsymbol{\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}}$$

$$\textbf{or } \boldsymbol{\tau_{max} = \frac{16}{\pi}\sqrt{\frac{F^2}{64{d_1}^4} + \frac{T^2}{d_1^6}}}$$

The screw-thread form is complicated from an analysis viewpoint. Remember the origin of the tensile-stress area A_t , which comes from experiment. A power screw lifting a load is in compression and its thread pitch is *shortened* by elastic deformation. Its engaging nut is in tension and its thread pitch is *lengthened*. The engaged threads cannot share the load equally. Some experiments show that the first engaged thread carries 0.38 of the load, the second 0.25, the third 0.18, and the seventh is free of load. In estimating thread stresses by the equations above, substituting 0.38 F for F and setting n_t to 1 will give the largest level of stresses in the thread-nut combination.

Ham and Ryan (1932) showed that the coefficient of friction in screw threads is independent of axial load, practically independent of speed, decreases with heavier lubricants, shows little variation with combinations of materials, and is best for steel on bronze. Sliding coefficients of friction in power screws are about 0.10–0.15. The following table shows safe bearing pressures on threads, to protect the moving surfaces from abnormal wear.

The next table shows the coefficients of sliding friction for common material pairs. And the after next table shows coefficients of starting and running friction for common material pairs.

Table Screw p_b *Source:* H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook,* 2nd ed., McGraw-Hill, New York, 2006.

Screw Material	Nut Material	Safe Bearing Pressure, p_b (psi)	Notes
Steel	Bronze	2500–3500	Low speed
Steel	Bronze	1600–2500	≤10 fpm
	Cast iron	1800–2500	≤8 fpm
Steel	Bronze	800–1400	20–40 fpm
	Cast iron	600–1000	20–40 fpm
Steel	Bronze	150–240	≥50 fpm

Table: Screw-Nut Material Combination and Safe Bearing Pressure

Application	Material		Safe Bearing Pressure	Rubbing Velocity at Mean Diameter
	Screw	Nut	(MPa)	m/min
Hand Press	Steel Steel	Bronze C.I	17.5-24.5 12.5-17.5	Well lubricated Low Velocity
Screw Jack	Steel Steel	C.I Bronze	12.5-17.5 10.5-17.5	Velocity < 2.5 Velocity < 3.0
Hoisting Machine	Steel Steel	C.I Bronze	4.0-7.0 35.0-100.0	6-12 6-12
Lead Screw	Steel	Bronze	10.5-17.0	> 15.0

Table : Coefficients of Friction *f* for Threaded Pairs *Source:* H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook*, 2nd ed., McGraw-Hill, New York, 2006

Screw Material	Nut Material			
	Steel	Bronze	Brass	Cast Iron
Steel, dry	0.15–0.25	0.15–0.23	0.15–0.19	0.15–0.25
Steel, machine oil	0.11–0.17	0.10–0.16	0.10–0.15	0.11–0.17
Bronze	0.08–0.12	0.04–0.06	—	0.06–0.09

Table: Thrust-Collar Friction Coefficients *Source:* H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook*, 2nd ed., McGraw-Hill, New York, 2006.

Combination	Running	Starting
Soft steel on cast iron	0.12	0.17
Hard steel on cast iron	0.09	0.15
Soft steel on bronze	0.08	0.10
Hard steel on bronze	0.06	0.08

Design procedure of Power screw:

1.Find **d** by considering that power screw is under pure compression as staring estimate .

2. Find the standard dimensions that close to the above estimate.

3. Find $\mathbf{T_L}$ to rotate the power screw against friction at the thread area and the collar torque **($\mathbf{T_c}$**) and the total torque ($\mathbf{T_t}$ **)** will be their sum.

4. Find the shear stress according the above torque.

5. Find the actual compression stress for the selected dimension (note: use the root cross sectional area of the screw.

6. Find the principal stresses and if max. shear theory is used then

$$\boldsymbol{\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2} \leq \tau_p}$$

Note here if the applied shear found to be above the permissible shear then the size must be enlarged and the starting again from step **2**.

7. Find the efficiency of the power screw

8. Find the active number of the threads of the nut **(n)** from the permissible pressure between the screw and the nut.

$$\boldsymbol{pb = \frac{F}{\frac{\pi}{4}\left(d^2 - d_1^2\right)n}}$$

9. Find the number of the threads of the nut **(n)** from shear stress of the nut

$$\boldsymbol{\tau_{nut} = \frac{F}{\pi\, d\, p\, n}}$$

10. Choose the number of the threads **(n)** of the nut as the larger between the two values.

11. Find the height of the nut **(H)** as:

$$\mathbf{H = (n+2)\ p}$$

12. Find the outer diameter of the nut D_{nut} from:

$$\boldsymbol{\sigma_{tp} = \frac{F}{\frac{\pi}{4}\left(D_{nut}^2 - d^2\right)}}$$

13. Find the other dimensions of the nut that depends in the design of the nut.

14. Show the mechanism of the collar and if it large use the theory of friction disk.

15. Design the mechanism of the handle that provide the working torque. Find the dimension of the handle. Assume that person can apply around 300-400 N by his hand.

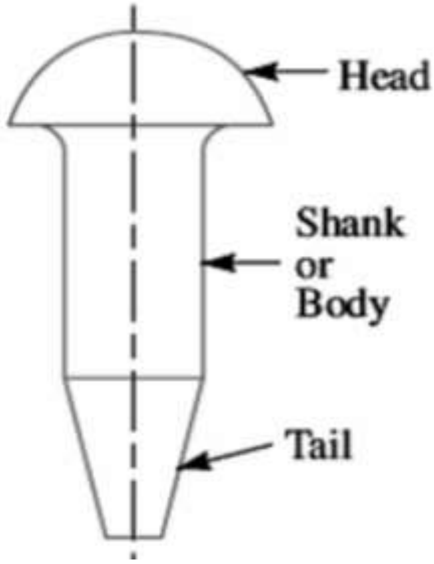
16. Check screw for buckling (use Euler's formula for long column and Johnson formula for short column).

17. Find The other dimensions of body and other accessories (according to the design of them).

18. Check for self-locking conditions.

Rivets and Riveted Joints:

The rivets are used to make permanent fastening between the two or more plates such as in structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals. A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called shank or body and lower portion of shank is known as tail.



Methods of Riveting

The function of rivets in a joint is to make a connection that has strength and tightness. The strength is necessary to prevent failure of the joint. The tightness is necessary in order to contribute to strength and to prevent leakage as in a boiler or in a ship hull (The frame or body of ship). When two plates are to be fastened together by a rivet as shows below, the holes in the plates are punched and reamed or drilled. Punching is the cheapest method and is used for relatively thin plates and in structural work. Since punching injures the material around the hole, therefore drilling is used in most pressure-vessel work. The creation of head by process of upsetting is shown in the following figure The upsetting of the cylindrical portion of the rivet can be done cold or hot. When diameter of rivet is 12 mm or less, cold upsetting can be done. For larger diameters the rivet is first heated to light red and inserted. The head forming immediately follows. The rivet completely fills the hole in hot process. Yet it must be understood that due to subsequent cooling the length reduces and diameter decreases. The reduction of length pulls the heads of rivet against plates and makes the joint slightly stronger. The reduction of diameter creates clearance between the inside of the hole and the rivet. Such decrease in length and diameter does not occur in cold worked rivet.

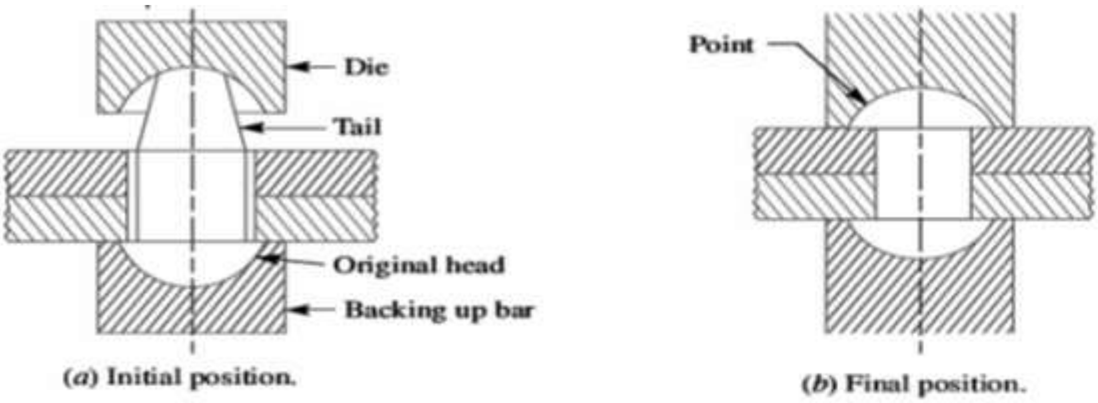


Figure: Methods of Riveting

Material of Rivets

The material of the rivets must be tough and ductile. They are usually made of steel (low carbon steel or nickel steel), brass, aluminum or copper, but when strength and a fluid tight joint is the main consideration, then the steel rivets are used. The rivets for general purposes shall be manufactured from steel conforming to the following Indian Standards:

1. IS: 1148–1982 (Reaffirmed 1992) – Specification for hot rolled rivet bars (up to 40 mm diameter) for structural purposes; or

2. IS: 1149–1982 (Reaffirmed 1992) – Specification for high tensile steel rivet bars for structural purposes.

3. The rivets for boiler work shall be manufactured from material conforming to IS: 1990 – 1973 (Reaffirmed 1992) – Specification for steel rivets and stay bars for boilers.

Manufacture of Rivets

The rivets may be made either by cold heading or by hot forging.

1. If rivets are made by the cold heading process, they heat treated so that the stresses set up in the cold heading process are eliminated.

2. If they are made by hot forging process, care shall be taken to see that the finished rivets cool gradually.

Note: when the diameter of rivet is 12 mm or less generally cold riveting is adopted.

Types of Rivets

1. Button Head
2. Counter sunk Head
3. Oval counter Head
4. Pan Head
5. Conical Head

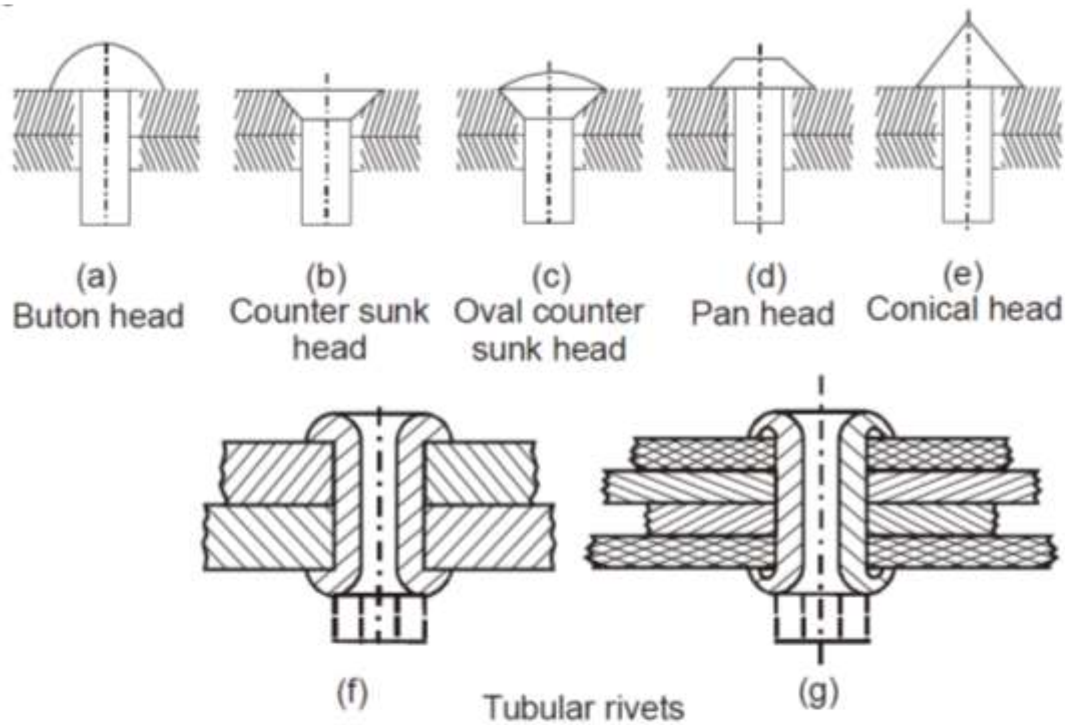


Figure: Different Types of Rivet Heads

Types of Riveted Joints

1. According to purpose
2. According to position of plates connected
3. According to arrangement of rivets

1. According to purpose:

a) Strong Joints: In these Joints strength is the only criterion (e.g.: Beams, Trusses and Machine Joints).

b) Tight joints: These joints provide strength as well as are leak proof against low pressure (e.g. Reservoir, Containers and tanks).

c) Strong-Tight Joints: These are the joints applied in boilers and pressure vessels and ensure both strength and leak proofness.

2. According to position of plates:

a) Lap Joint: A lap joint is that in which one plate overlaps the other and the two plates are then riveted together.

b) Butt Joint: A butt joint is that in which the main plates are touching each other and a cover plate (i.e. Strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with the main plates. Butt joints are of the following two types:

I) In a single strap butt joint, the edges of the main plates butt against each other and only one cover plate is placed on one side of the main plates and then riveted together.

II) In a double strap butt joint, the edges of the main plates butt against each other and two cover plates are placed on both sides of the main plates and then riveted together.

3. According to arrangement of rivets:

a) A **single riveted joint** is that in which there is a single row of rivets in a lap joint as shown in following figure and there is a single row of rivets on each side in a butt joint as shown in part (a) of the figure.

b) A **double riveted joint** is that in which there are two rows of rivets in a lap joint as shown in the following figure and there are two rows of rivets on each side in a butt joint as shown in parts (b and c) of the figure.

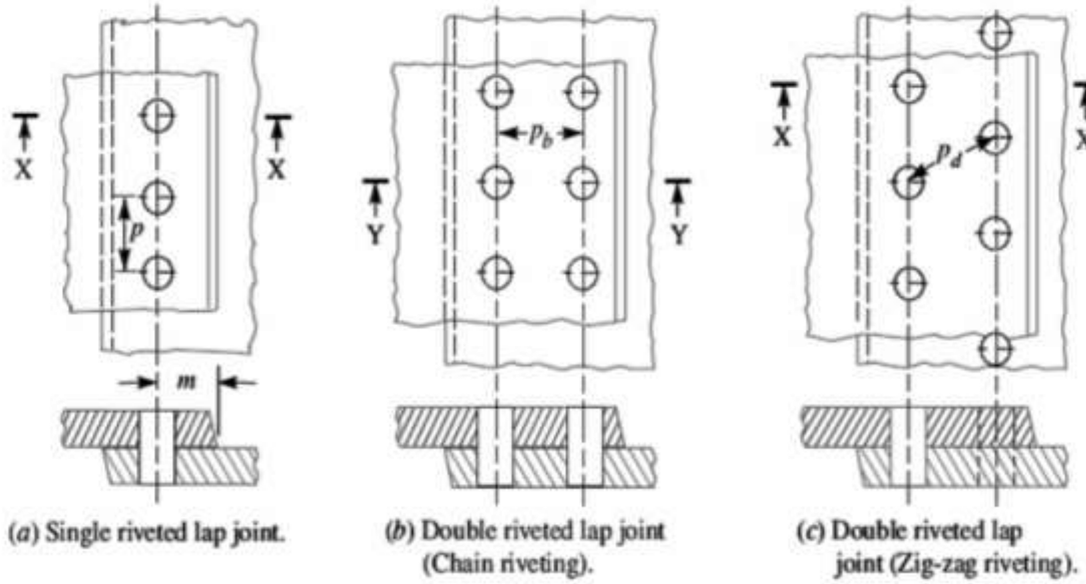


Figure Types of riveted joints according to arrangement of rivets

Important terms of Riveted joints:

1. Pitch (p): The Distance between two adjacent rivet holes in a row.

2. Back pitch (p_b): The Distance between two adjacent rows of rivets.

3. Diagonal pitch (p_d): The smallest distance between centers of two rivet holes in adjacent rows of Zig-Zag riveted joints.

4. Margin (m): It is the distance between center of a rivet hole and nearest edge of the plate.

5. The plates to be jointed are often of the same **thickness** and their thickness is denoted by ***t***. However, if the thicknesses are different, the lower one will be denoted by t_1.
6. The thickness of the cover plate (also known as strap) in a butt joint will be denoted as t_c.
7. The rivet **hole diameter** is denoted by ***d***. This diameter is normally large than the diameter of the rivet shank which is denoted by d_1.
A problem of designing of a riveted joint involves determinations of p, p_b, p_d, m, t, t_c and ***d***, depending upon type of the joint.

Modes of Failures of a Riveted Joint
1. Tearing of the plate at the section weakened by holes: Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.

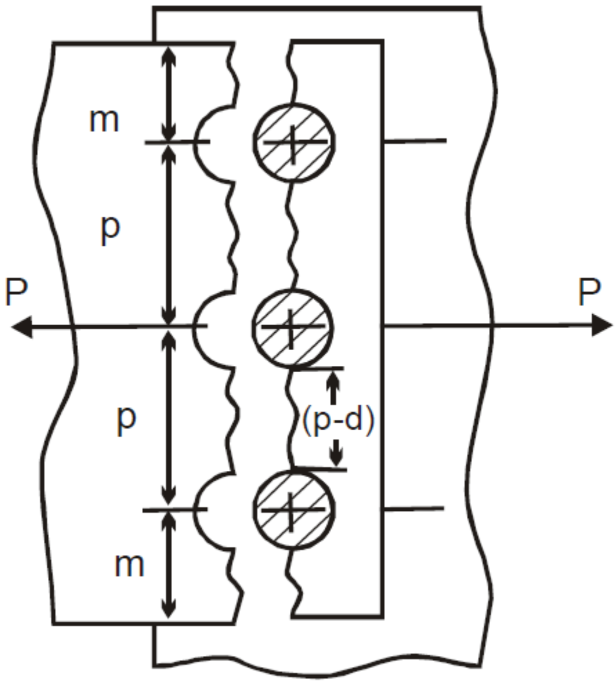
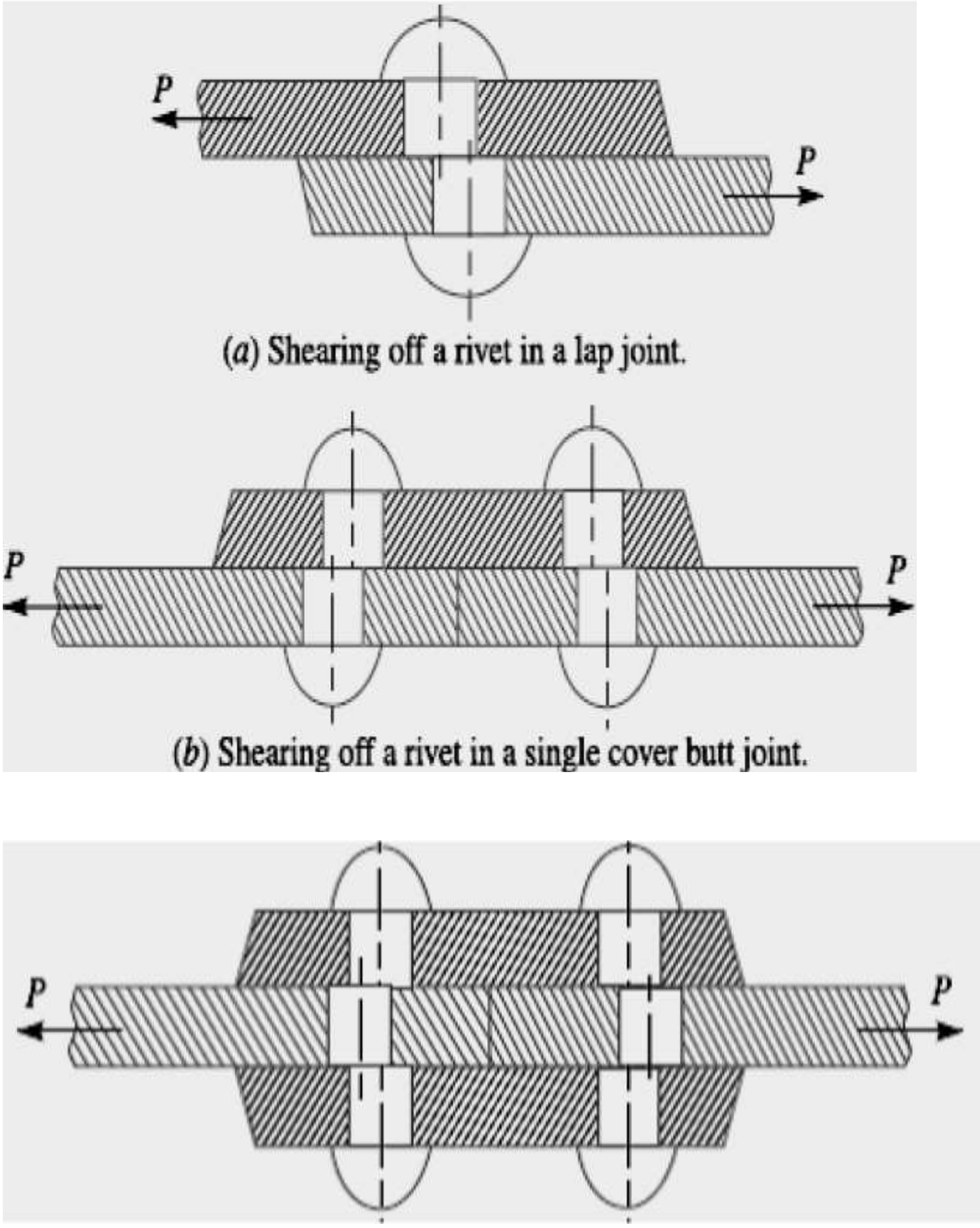


Figure : Modes of Failures of a single lap Riveted Joint (**Tearing of the plate**).

2. Shearing of Rivet: The failure will occur when all the rivets in a row shear off simultaneously. Considers the strength provided by the rivet against this mode of failure, one consider number of rivets in a pitch length which is obviously one. Further, in a lap joint failure due to shear may occur only along one section of rivet as shown in Figure (a). However, in case of double cover butt joint failure may take place along two sections in the manner shown in Figure (c).

3. Crushing of Plate and Rivet: Due to rivet being compressed against the inner surface of the hole, there is a possibility that either the rivet or the hole surface may be crushed. The area, which resists this action, is the projected area of hole or rivet on diametral plane.

4. Shearing of Plate Margin near the Rivet Hole : The following Figure shows this mode of failure in which margin can shear along planes *ab* and *cd*. If the length of margin is *m*, the area resisting this failure is, 2***mt***.



(*a*) Shearing off a rivet in a lap joint.

(*b*) Shearing off a rivet in a single cover butt joint.

(c) Shearing off a rivet in a double cover butt joint.

Figure : Modes of Failures of a Riveted Joint (**Shearing of Rivet**).

In writing down the above equations for strength of the joint certain assumptions have been made. It is worthwhile to remember them. Most importantly it should be remembered that most direct stresses have been assumed to be induced in rivet and plate which may not be the case. However, ignorance of actual state of stress and its replacement by most direct stress is compensated by lowering the permissible values of stresses σ_{tp}, τ_p and σ_{bp}, i.e. by increasing factor of safety. The assumptions made in calculations of strengths of joint are :

(a) The tensile load is equally distributed over pitch lengths.

(b) The load is equally distributed over all rivets.
(c) The bending of rivets does not occur.
(d) The rivet holes do not produce stress concentration. The plate at the hole is not weakened due to increase in diameter of the rivet during second head formation.
(e) The crushing pressure is uniformly distributed over the projected area of the rivet.
(f) Friction between contacting surfaces of plates is neglected.

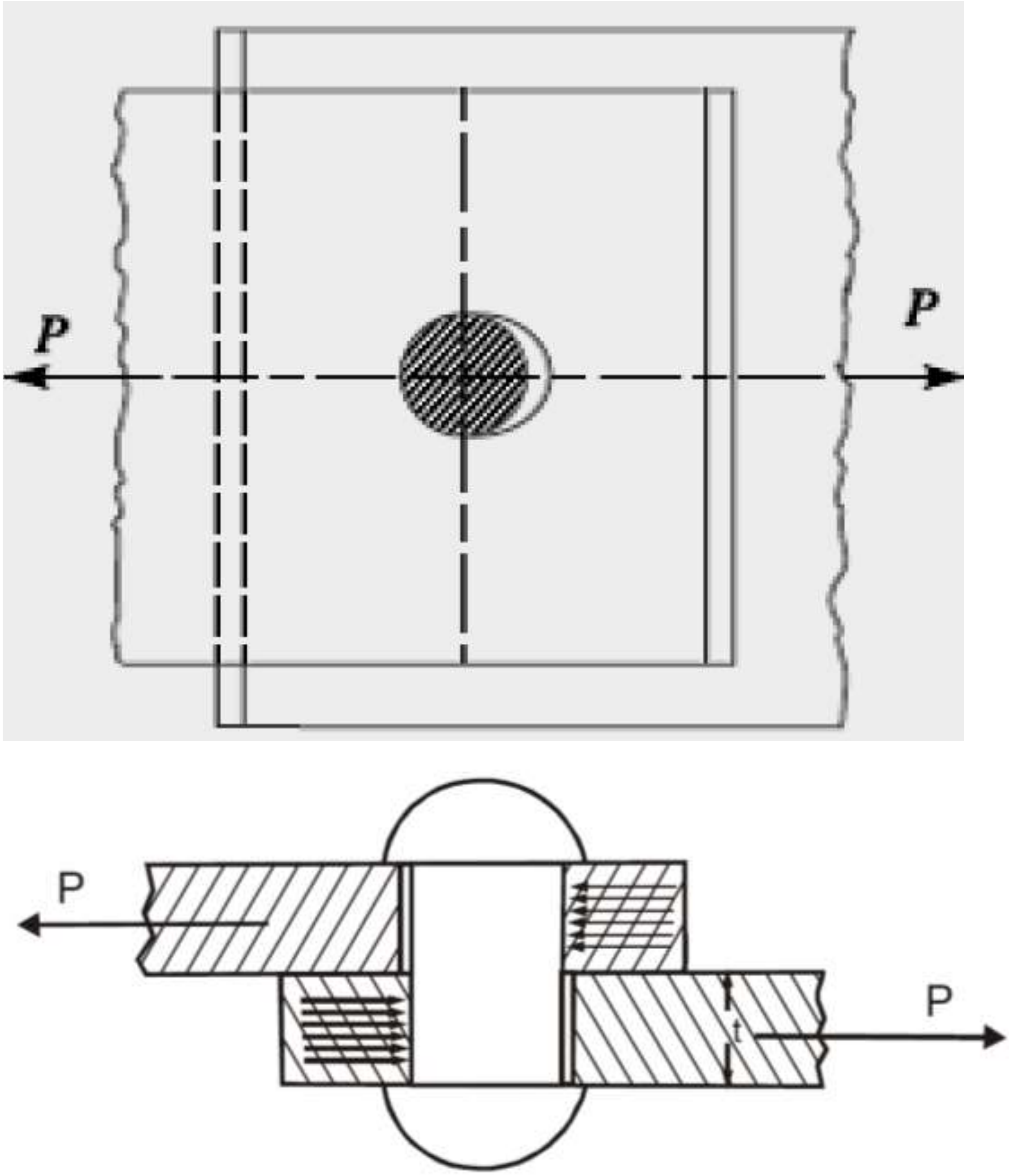


Figure : Modes of Failures of a Riveted Joint **(Crushing of Plate and Rivet).**

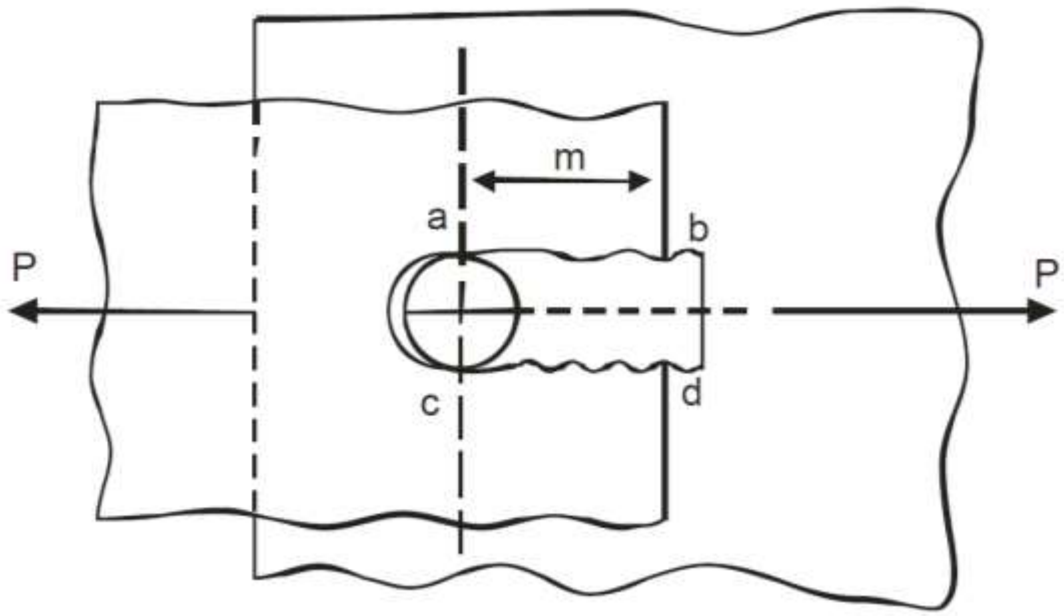


Figure : Modes of Failures of a Riveted Joint **(Shearing of Plate Margin near the Rivet Hole)**

Table : Standard Rivet Hole and Rivet Diameters

***d* (mm)**	13	15	17	19	21	23	25	28.5	31.5	34.5	37.5	41	44
***d_1* (mm)**	12	14	16	18	20	22	24	27	30	33	36	39	42

Table : Load sharing factor in multiple riveting

No. of Rivet	**Load Sharing in Multiple Riveting**	
	Max. Fraction	**Average**
3	0.353	0.33
4	0.29	0.25
5	0.26	0.2
6	0.24	0.166

Eccentric Loaded Riveted Joint

When the line of action of the load does not pass through the centroid of the rivet system and thus all rivets are not equally loaded, then the joint is said to be an eccentric loaded riveted joint, as shown in the following Figure. The eccentric loading results in secondary shear caused by the tendency of force to twist the joint about the centre of gravity in addition to direct shear or primary shear.

Let *P* = Eccentric load on the joint, and *e* = Eccentricity of the load *i.e.* the distance between the line of action of the load and the centroid of the rivet system *i.e. G*.

The following procedure is adopted for the design of an eccentrically loaded riveted joint.

1. First of all, find the centre of gravity *G* of the rivet system. Let A = Cross-sectional area of each rivet, x_1, x_2, x_3 etc. = Distances of rivets from O-Y, and y_1, y_2, y_3 etc. = Distances of rivets from O-X. We know that

$$\bar{X} = \frac{(AX_1 + AX_2 + AX_3 + AX_4 + AX_5 + AX_6 + AX_7 + ..)}{(nA)} = \frac{(X_1 + X_2 + X_3 + ..)}{(n)}$$

$$\bar{y} = \frac{(Ay_1 + Ay_2 + Ay_3 + Ay_4 + Ay_5 + Ay_6 + Ay_7 + ..)}{(nA)} = \frac{(y_1 + y_2 + y_3 + ..)}{(n)}$$

Where;

n = Number of Rivet and note that all of rivets are with the same size

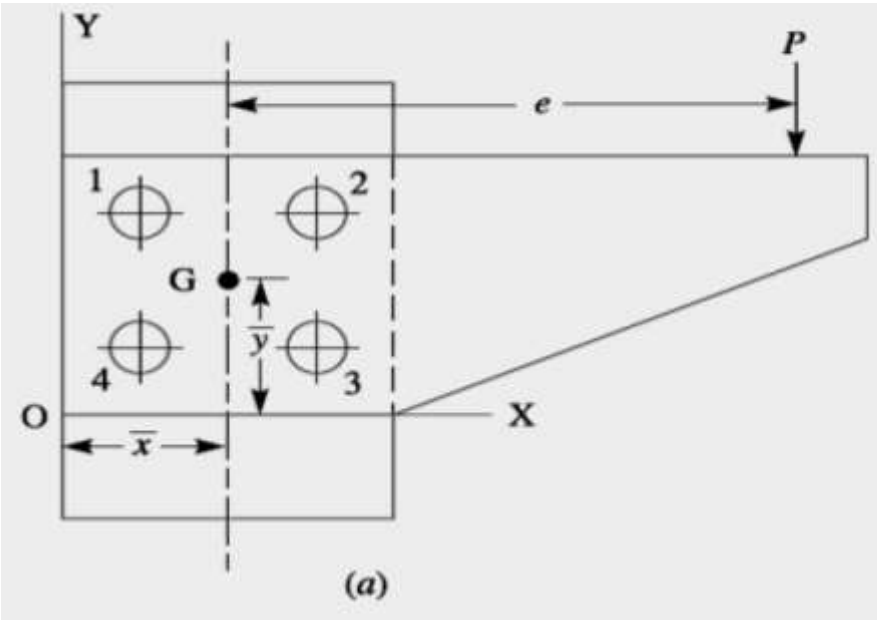


Figure Eccentric Loaded Riveted Joint

2. Introduce two forces P_1 and P_2 at the centre of gravity 'G' of the rivet system. These forces are equal and opposite of P as shown in the figure.

3. Assuming that all the rivets are of the same size, the effect of P_1 = P is to produce direct shear load on each rivet of equal magnitude. Therefore, direct shear load on each rivet, $P_s = P/n$, acting parallel to the load P.

4. The effect of P_2 = P is to produce a turning moment of magnitude P × e which tends to rotate the joint about the centre of gravity 'G' of the rivet system in a clockwise direction. Due to the turning moment, secondary shear load on each rivet is produced. In order to find the secondary shear load, the following two assumptions are made:

a). The secondary shear load is proportional to the radial distance of the rivet under consideration from the centre of gravity of the rivet system.

b). The direction of secondary shear load is perpendicular to the line joining the centre of the rivet to the centre of gravity of the rivet system.

Let F_1, F_2, F_3 ... = Secondary shear loads on the rivets 1, 2, 3...etc.

l_1, l_2, l_3 ... = Radial distance of the rivets 1, 2, 3 ...etc. from the centre of gravity 'G' of the rivet system.

∴ From assumption (a),

$$F_1 \; \alpha \; l_1 \; ; F_2 \; \alpha \; l_2 \;\; and \; so \; on$$

$$\frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{F_3}{l_3} \;\; = \cdots$$

$$F_2 = F_1 \frac{l_2}{l_1}, and \; F_3 = F_1 \frac{l_3}{l_1} \;\; and \; so \; on$$

We know that the sum of the external turning moment due to the eccentric load and of internal resisting moment of the rivets must be equal to zero.

$$\therefore \; P.\,e = F_1.l_1 + F_2.l_2 + F_3.l_3 + \ldots$$

$$= F_1 \times l_1 = F_1 \times \frac{l_2}{l_1} \times l_2 + F_1 \times \frac{l_3}{l_1} \times l_3 \; + \cdots$$

$$= \; \frac{F_1}{l_1} \; [(l_1)^2 \; + (l_2)^2 \; + (l_2)^2 + \cdots$$

From the above expression, the value of F_1 may be calculated and hence F_2 and F_3 etc. are known. The direction of these forces are at right angles to the lines joining the centre of rivet to the centre of gravity of the rivet system, as shown in the figure. and should produce the moment in the same direction (i.e. clockwise or anticlockwise) about the centre of gravity, as the turning moment (P × e).

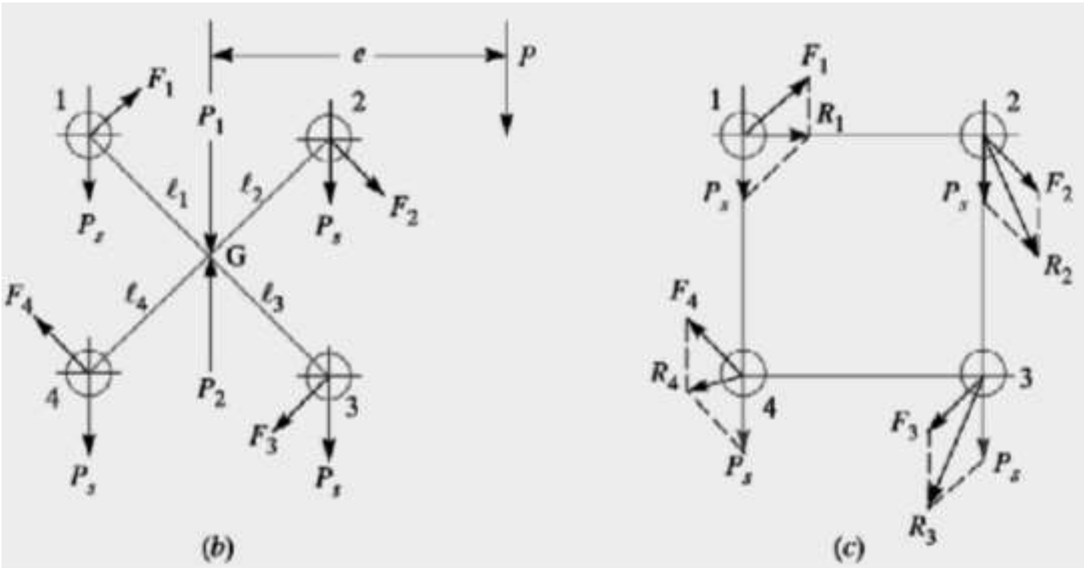


Figure Eccentric Loaded Riveted Joint analysis

5. The primary (or direct) and secondary shear load may be added vectorially to determine the resultant shear load (R) on each rivet as shown in the figure. It may also be obtained by using the relation

$$R = \sqrt{(P_2)^2 + \ F^2 \quad + 2P_s \times F \ \times \cos\theta}$$

θ = Angle between the primary or direct shear load (P_s) and secondary shear load (F). When the secondary shear load on each rivet is equal, then the heavily loaded rivet will be one in which the included angle between the direct shear load and secondary shear load is minimum. The maximum loaded rivet becomes the critical one for determining the strength of the riveted joint. Knowing the permissible shear stress (τ_p), the diameter of the rivet hole may be obtained by using the relation,
Maximum resultant shear load (R) = $(4/\pi) \times d^2 \times \tau$
From Table, the standard diameter of the rivet hole (d*)* and the rivet diameter may be specified, according to IS: 1929 – 1982 (Reaffirmed 1996).

Steps involving for solving the eccentricity Problems:

1. Firstly find the centre of gravity *G.*
2. Find Direct Shear load P_s.
3. Find Turning moment produced by the load P due to eccentricity e. (P x e).
4. Find Radial distance of the rivets (l_1, l_2, l_3, l_4…………).
5. Find Secondary shear loads on the rivets (F_1, F_2, F_3, F_4………..).
6. Find the Angle between the direct and secondary shear load of the rivets.
7. Resultant Shear load (R) on the rivets.
8. Find Diameter of rivet hole (d).
9. Then find the diameter of rivet (D_r) from the Design Data Book or the previous table.

Example: According to part (a) of the following figure, the distances between columns and rows of rivets are shown. Calculate the maximum shearing stress in rivets if the force *P* = 1kN note that each rivet is 5 mm in diameter. Note that all dimensions on the figure are in mm.

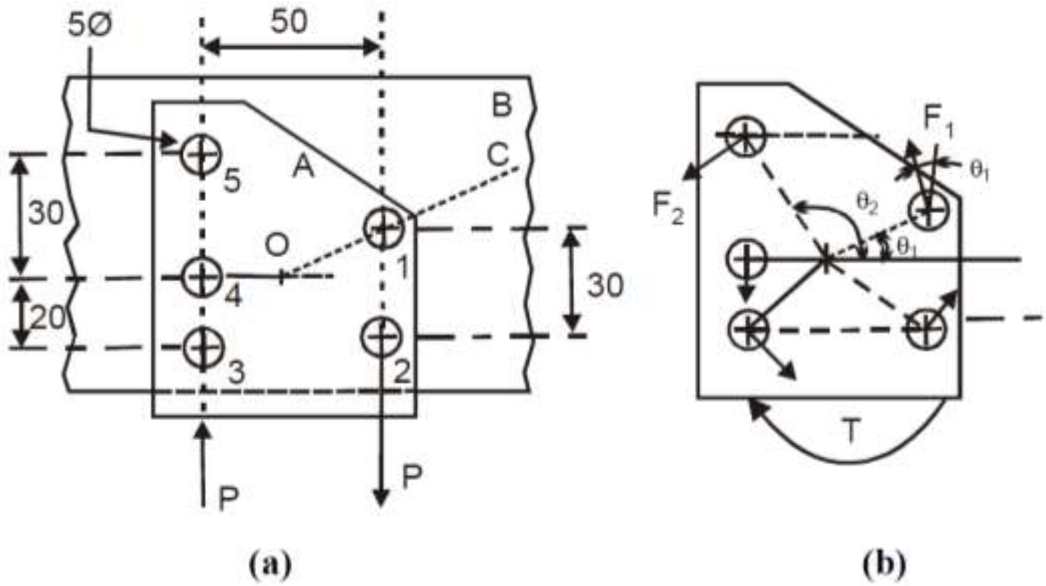


Figure : Torsional Loading And Eccentric Loading Of Riveted Joint

Solution (Analysis)

Plate *A* is riveted to structural element *B.* A torque is applied to the Plate *A.* The plate will rotate, of course by slight elastic amount, about some point as **O** in part (a) of the above Figure. It is not wrong to assume that any straight line such as **O-C** which passes through the centre of a rivet, remains straight before and after application of the torque. Then the deformation, hence strain and so the average shearing stress across the section of the rivet will be proportional to the distance between **O** and the centre of the rivet. Since the average shearing stress is equal to the shearing force divided by area of cross section of the rivet, the shearing force on the rivet will be proportional to the distance between **O** and centre of the rivet. The direction of this force will be perpendicular to the joining line.

The forces F_1, F_2, etc. on individual rivets are shown in part (b) of the Figure. For satisfying condition of equilibrium, components of forces in vertical direction should sum up to zero. If the forces F_1, F_2, etc. make angles θ_1, θ_2, etc. respectively with *y*-axis, then

$F_1 \cos\theta_1 + F_2\cos\theta_2 + \ldots\ldots + F_n \cos\theta_n = 0$

or . . . $\sum_{i=1}^{n} F_i \cos\theta = 0\ldots$ …… (i)

But $F_i = \tau_i A_i = (\tau_i / r_i)\, r_i A_i$

And since $\tau_i \propto r_i$ or $\tau_i = k\, r_i$

So, $F_i = k\, r_i A_i$ …… (ii)

Here k is a constant, τ_i = shearing stress in i^{th} rivet whose area of cross section is A_i and its centre is at a distance r_i from O.

Use (ii) and (i) to obtain

$$\sum_{i=1}^{n} r_i\, A_i \cos\theta_i = 0$$

See from Figure part (b) that $r_i \cos_i \theta_i = x$

$$k \sum_{i=1}^{n} x\, A_i = 0$$

Which is same as $\bar{x}\, A_t = 0$

Where $\bar{x}$ is the *x*-coordinate of centroid of all the rivet and sum of their areas of cross sections is A_t. And since neither k nor A_t is zero therefore, $\bar{x}$ =0. If then we consider sum of forces along *x*-axis we would arrive at the result $\bar{y}$ =0. This means that *0* is the point coinciding with the centroid of the rivet area system.

Solution (Numerical value)

The five rivets have been numbered as 1, 2, . . . , 5. Take centre of rivet 3 as origin and *x* and *y* axes along 3-2 and 3-5 respectively. Areas of all rivets is

$A = \frac{\pi}{4}(5)^2 = 19.64\ \text{mm}^2$. If $\bar{x}$ and $\bar{y}$ are the coordinates of the centroid, then

$$50A + 50A = 5A\,\bar{x}$$

Hence, $\bar{x} = 20$ mm

Also, $50A + 20A + 30A = 5A\,\bar{y}$, $\bar{y} = 20$ mm

Hence centroid is on the horizontal line through rivet 4. We can calculate various distances of rivet centers from centroid.

$$r_1 = \sqrt{10^2 + 30^2} = 10\sqrt{10}$$

$$r_2 = \sqrt{20^2 + 30^2} = 10\sqrt{13}$$

$$r_3 = \sqrt{20^2 + 20^2} = 10\sqrt{8}$$

$$r_4 = \sqrt{20^2 + 0} = 10 \times 2$$

$$r_5 = \sqrt{30^2 + 20^2} = 10\sqrt{13}$$

Now $F_1 = k\,r_1 = k\;10\sqrt{10}$, moment of F_1 about O,

$$M_1 = k\,r_1^2 = 1000\,k$$

$$F_2 = k\,r_2 = k\,10\sqrt{13},\quad M_2 = 1300\,k$$

$$\frac{F_1}{F_2} = \frac{r_1}{r_2} = \frac{\sqrt{10}}{\sqrt{13}} \quad \text{or} \quad F_1 = \sqrt{\frac{10}{13}}\,F_2$$

We can find each of F_1, F_2, F_3, F_4 and F_5 in terms of k or we can find each force in terms of F_2. We may like to choose F_2 because F_2 is grater than all other forces because r_2 is larger than all other r.

$$F_3 = \sqrt{\frac{8}{13}}\,F_2,\; F_4 = \frac{2}{\sqrt{13}}\,F_2,\; F_5 = F_2$$

Taking moments of all forces about O and equating with the applied moment of $50\,P = 50000$ N mm.

$$\sqrt{\frac{10}{13}}\,F_2\,10\sqrt{10} + F_2\,10\sqrt{13} + \sqrt{\frac{8}{13}}\,F_2\,10\sqrt{8} + \frac{2}{\sqrt{13}}\,F_2\,10 \times 2 + F_2\,10\sqrt{13} = 5 \times 10^4$$

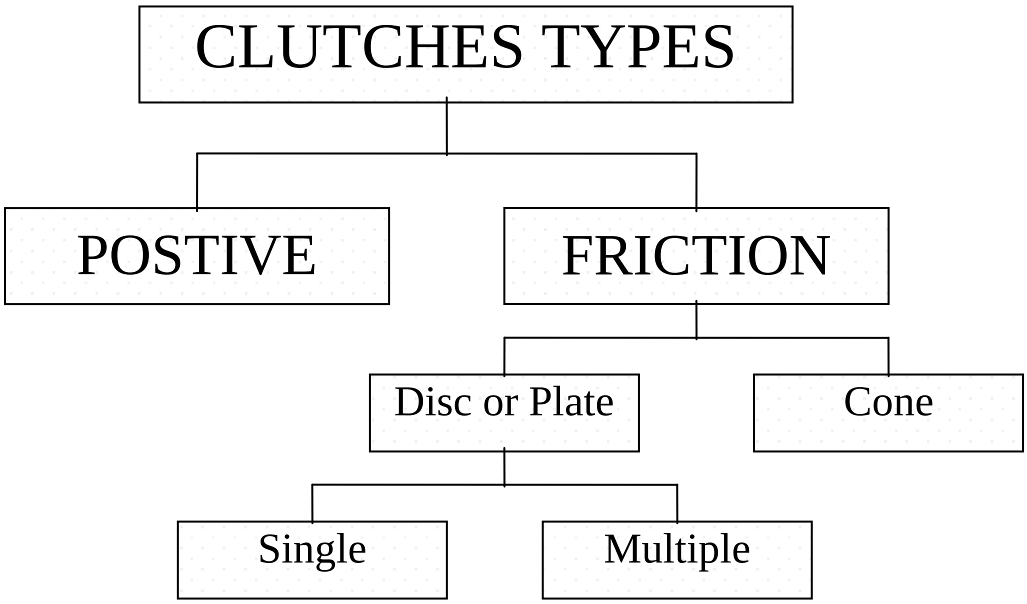
$$\frac{100F_2}{\sqrt{13}} + \frac{130F_2}{\sqrt{13}} + \frac{80F_2}{\sqrt{13}} + \frac{40F_2}{\sqrt{13}} + \frac{130F_2}{\sqrt{13}} = 5 \times 10^4$$

$$\therefore \qquad F_2 = \frac{500\sqrt{13}}{4.8} = 375.6 \text{ N}$$

$$\therefore \quad \text{Maximum shearing stress} = \frac{F_2}{A} = \frac{375.6}{19.64} = 19.12 \text{ N/mm}^2$$

This stress will be in rivets 2 and 5.

Clutches: A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped as will, without stopping the driving shaft. It is, also, a friction device which permits the connection and disconnection of shafts. Clutches could be either positive or friction type. In positive(Jaw) clutch the two shaft are rigidly connected and rotate at the same speed in "in" position and remain entirely disconnected in the "out" position. On the other hand friction clutches are gradually engaging until the fully engagement the two shaft rotate as one.



General notes:

1. For multiple disc clutch Torque most be multiplied by n, where n is the number of pairs of contact surfaces. If there are n_1 discs on the driving shaft and n_2 disc on the driven shaft then the number of the contact surfaces:

$$n = n_1 + n_2$$

2. In the case of new clutch use uniform pressure. In the case of old clutch use uniform wear.

3. Recommended $R_1/R_2 \approx 0.6\text{-}0.8$ and if (b = disc width) then

$$b/r_m = (R_1-R_2)/\ 0.5(R_1-R_2) \approx 0.22\text{-}0.5$$

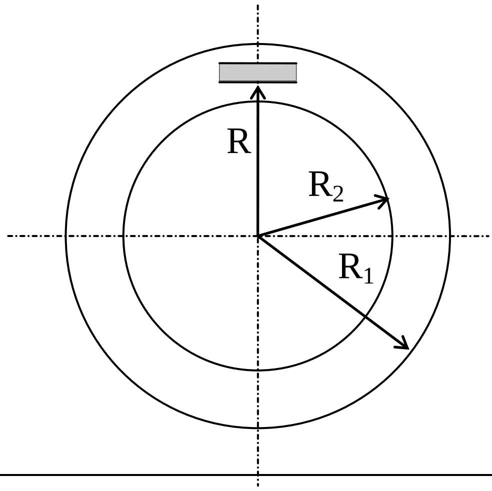
4. The design torque is higher than the motor torque because of engagement factor(β).

$$T_{design} = \beta\ T_{motor}$$

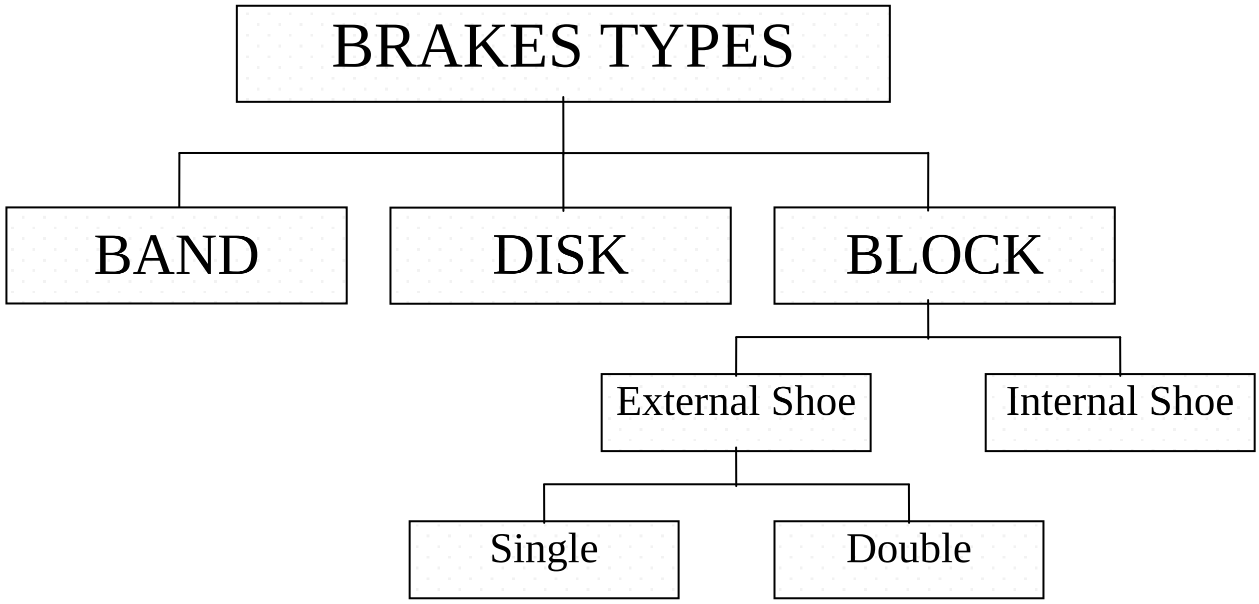
β can be fond in the following table:

Application	β
Metal-cutting machine tool	1.26-1.5
Tractor	2 - 2.5
Automobile	1.2-1.5
Crane machine	> 1.15

R_2: Inner radius of the clutch
R_1: Outer radius of the clutch
T: Torque transmitted
P: Axial pressure to held surfaces
μ: Coefficient of friction



Brakes: Are machine element that absorb either kinetic or potential energy in the process of slowing down or stopping a moving part. The absorbed energy is dissipated as heat. Brake capacity depends upon the unit pressure between the braking surfaces, the coefficient of friction, and the ability of the brake to dissipate heat equivalent to the energy being absorbed. The performance of brakes is similar to that of clutches except that clutches connect one moving part to another moving part, whereas brakes connect a moving part to a frame.



Power Transmission Systems

In the design of a power transmission, you would typically know the following:

• ***The nature of the driven machine:*** It might be a machine tool in a factory that cuts metal parts for engines; an electric drill used by professional carpenters or home craft workers; the axle of a farm tractor; the propeller shaft of a turbojet for an airplane; the propeller shaft for a large ship; the wheels of a toy train; a mechanical timing mechanism; or any other of the numerous products that need a controlled-speed drive.

• ***The level of power to be transmitted:*** From the examples just listed, the power demanded may range from thousands of horsepower for a ship, hundreds of horsepower for a large farm tractor or airplane, or a few watts for a timer or a toy.

• ***The rotational speed of the drive motor or other prime mover:*** Typically the prime mover operates at a rather high speed of rotation. The shafts of standard electric motors rotate at about 1200. 1800. or 3600 revolutions per minute (rpm). Automotive engines operate from about 1000 to 6000 rpm. Universal motors in some hand tools (drills, saws, and routers) and household appliances (mixers, blenders, and vacuum cleaners) operate from 3500 to 20 000 rpm. Gas turbine engines for aircraft rotate many thousands of rpm.

• ***The desired output speed of the transmission:*** This is highly dependent on the application. Some gear motors for instruments rotate less than 1.0 rpm. Production machines in factories may run a few hundred rpm. Drives for assembly conveyors may run fewer than 100 rpm. Aircraft propellers may operate at several thousand rpm.

The designer of a power transmission system must do the following:

• Choose the type of power transmission elements to be used: gears, belt drives, chain drives, or other types. In fact, some power transmission systems use two or more types in series to optimize the performance of each.

• Specify how the rotating elements are arranged and how the power transmission elements are mounted on shafts.

• Design the shafts to be safe under the expected torques and bending loads and properly locate the power transmission elements and the bearings. It is likely that the shafts will have several diameters and special features to accommodate keys, couplings, retaining rings, and other details. The dimensions of all features must be specified, along with the tolerances on the dimensions and surface finishes.

• Specify suitable bearings to support the shafts and determine how they will be mounted on the shafts and how they will be held in a housing.

• Specify keys to connect the shaft to the power transmission elements; couplings to connect the shaft from the driver to the input shaft of the transmission or to connect the output shaft to the driven machine; seals to effectively exclude contaminants from entering the transmission; and other accessories.

• Place all of the elements in a suitable housing that provides for the mounting of all elements and for their protection from the environment and their lubrication.

General notes:

1. It is very common to find a power transmission system interposed between the driving prime movers, e.g. electric motor, engine, turbine and etc. and driven machine.

2. In many cases power will not transmitted directly from the driving machine. Velocity change, velocity control, torque change, many output for one driving machine, velocity direction and safety consideration are examples for the intermediate elements.

3. Types of a power transmission system are include electrical, hydraulic, pneumatic and mechanical means like friction or mesh.

4. The following table shows the main characteristics of different transmission systems:

Table: Power transmission systems characteristics.

No.	**Characteristics**	**System(s)**
1	Controlled power supply	Electrical, Pneumatic
2	Transmission of power over large distance	Electrical
3	Accumulation of power	Hydraulic
4	Step-by-step velocity change over a wide range	Electrical, Mechanical (friction type)
5	Step-less change of velocity over a wide range	Electrical, Mechanical (both friction and mesh type)
6	Accurate velocity ratio	Mechanical (mesh type)
7	High velocities of rotation	Electrical, Pneumatic
8	No effect of ambient temperature	Electrical, Mechanical (mesh type)
9	Easy control (Remote or Automatic	Electrical

5. Mechanical drives classified:

A. According to the mode of transmission as :

Friction type

- Disk
- Belt (flexible)

Mesh type

- Chain (flexible)
- Gear

B. According to change of velocity ratio

- No change
- Step-by-step change
- Steplees

C. Position of the shaft

- Parallel
- Intersecting
- Skew

6. The following table shows a comparison of different Mechanical drives for 75 KW and Velocity Ratio 1000:25:

No.	**Drive**	**Centre Distance (mm)**	**Face Width (mm)**	**Weight (N)**	**Cost**
1.	Flat Belt	5000	350	5000	106
2.	Flat Belt with idler pulley	2300	250	5500	125
3.	V-Belt	1800	130	5000	100
4.	Chain	830	360	5000	140
5.	Toothed Gear	280	160	6000	165
6.	Worm and Gear	280	60	4500	125

7. The following table shows the main characteristics of Mechanical drives:

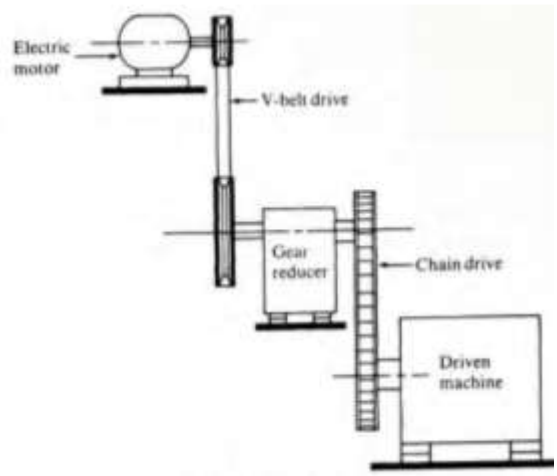
Table: Characteristics Of Mechanical Drives

Drive	Transmitted Power (kW)	Peripheral speed (m/sec)	Speed Ratio	Efficiency %
1. Flat Belt	≤ 100 (1500)*	5—30 (100)	≤4 (10)	80-92
2. V-Belt	≤ 50(300)	5—30	≤7 (15)	92-98
3. Chain	≤200 (5000)	≤25	≤15	94-98
4. Straight Toothed Gear	≤10,000	≤25	≤6,(10)	92-99
5. Helical Toothed Gear	≤50,000	≤25 (150)	≤7 (20)	94-99
6. Worm and Gear	≤100	≤35 (worm)	≤8—100 (1000)	10-98

* Quantities in parentheses are highest known values

8. Power transmission Example **(Gear-type speed reducer [Mott, 2004])**:

The following figure shows a typical industrial application of these elements combined with a gear-type speed reducer. This application illustrates where belts, gear drives, and chains are each used to best advantage. Rotary power is developed by the electric motor, but motors typically operate at too high a speed and deliver too low a torque to be appropriate for the final drive application. Remember, for a given power transmission, the torque is increased in proportion to the amount that rotational speed is reduced. So, some speed reduction is often desirable. The high speed of the motor makes belt drives somewhat ideal for that first stage of reduction. A smaller drive pulley is attached to the motor shaft, while a larger diameter pulley is attached to a parallel shaft that operates at a correspondingly lower speed. Pulleys for belt drives are also called *sheaves*. However, if very large ratios of speed reduction are required in the drive, gear reducers are desirable because they can typically accomplish large reductions in a rather small package. The output shaft of the gear-type speed reducer is generally at low speed and high torque. If both speed and torque are satisfactory for the application, it could be directly coupled to the driven machine. However, because gear reducers are available only at discrete reduction ratios, the output must often be reduced more before meeting the requirements of the machine. At the low-speed, high-torque condition, chain drives become desirable. The high torque causes high tensile forces to be developed in the chain. The elements of the chain are typically metal, and they are sized to withstand the high forces. The links of chains are engaged in toothed wheels called .*sprockets* to provide positive mechanical drive, desirable at the low-speed, high-torque conditions. In general, belt drives are applied where the rotational speeds are relatively high, as on the first stage of speed reduction from an electric motor or engine. The linear speed of a belt is usually 10 to 30 m/sec. which



results in relatively low tensile forces in the belt. At lower speeds, the tension in the belt becomes too large for typical belt cross sections, and slipping may occur between the sides of the belt and the sheave or pulley that carries it. At higher speeds, dynamic effects such as centrifugal forces, belt whip, and vibration reduce the effectiveness of the drive and its life. A speed of 20 m/sec is generally ideal. Some belt designs employ high-strength, reinforcing strands and a cogged design that engages matching grooves in the pulleys to enhance their ability to transmit the high forces at low speeds. These designs compete with chain drives in many applications.

As the designer, you must decide what type and size of belt drive to use and what the speed ratio between the driving and the driven sheave should be. How is the driving sheave attached to the motor shaft? How is the driven sheave attached to the input shaft of the gear reducer? Where should the motor be mounted in relation to the gear reducer, and what will be the resulting center distance between the two shafts? What speed reduction ratio will the gear reducer provide? What type of gear reducer should be used: helical gears, a worm and worm-gear drive, or bevel gears? How much additional speed reduction must the chain drive provide to deliver the proper speed to the driven shaft? What size and type of chain should be specified? What is the center distance between the output of the gear reducer and the input to the chopper? Then what length of chain is required? Finally, what motor power is required to drive the entire system at the stated conditions?

Flexible Mechanical Drives: Belt and Chain

This chapter will help you learn to identify the typical design features of commercially available belt and chain drives. You will be able to specify suitable types and sizes to transmit a given level of power at a certain speed and to accomplish a specified speed ratio between the input and the output of the drive. Installation considerations are also described so that you can put your designs into successful systems. Belts and chains are the major types of flexible power transmission elements. Belts operate on sheaves or pulleys, whereas chains operate on toothed wheels called *sprockets.*

Belts, ropes, chains, and other similar elastic or flexible machine elements are used in conveying systems and in the transmission of power over comparatively long distances. In many cases their use simplifies the design of a machine and substantially reduces the cost. In addition, since these elements are elastic and usually quite long, they play an important part in absorbing shock loads and in damping out and isolating the effects of vibration. This is an important advantage as far as machine life is concerned. Most flexible elements do not have an infinite life. When they are used, it is important to establish an inspection schedule to guard against wear, aging, and loss of elasticity. The elements should be replaced at the first sign of deterioration.

The mechanism of working of the belt:

A belt is a flexible power transmission element that seats tightly on a set of pulleys or sheaves. The following Figure shows the basic layout. When the belt is used for speed reduction, the typical case, the smaller sheave is mounted on the high-speed shaft, such as the shaft of an electric motor. The larger sheave is mounted on the driven machine. The belt is designed to ride around the two sheaves without slipping. The belt is installed by placing it around the two sheaves while the center distance between them is reduced. Then the sheaves are moved apart, placing the belt in a rather high initial tension. When the belt is transmitting power, friction causes the belt to grip the driving sheave, increasing the tension in one side, called the "tight side," of the drive. The tensile force in the belt exerts a tangential force on the driven sheave, and thus a torque is applied to the driven shaft. The opposite side of the belt is still under tension, but at a smaller value. Thus, it is called the "slack side."

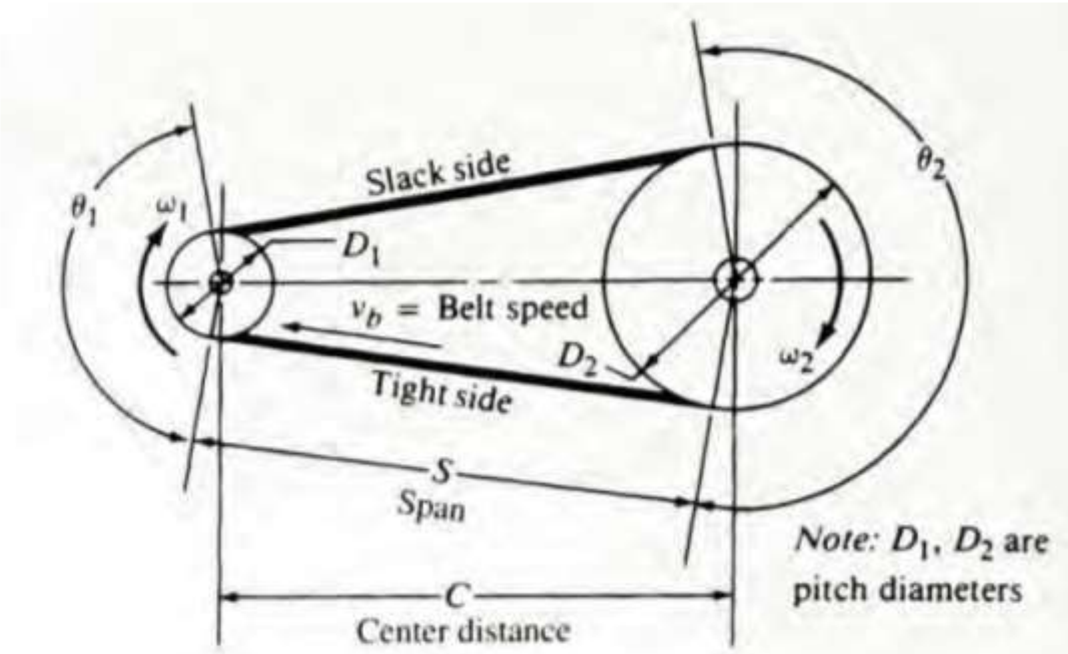


Figure : Basic belt drive geometry

Types of belt drives:

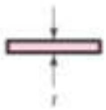
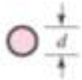
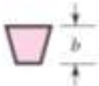
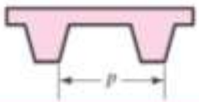
Many types of belts are available: flat belts, grooved or cogged belts, standard V-belts, double-angle V-belts. and others.

The flat belt is the simplest type, often made from leather or rubber-coated fabric. The sheave surface is also flat and smooth, and the driving force is therefore limited by the pure friction between the belt and the sheave. Some designers prefer flat belts for delicate machinery because the belt *will* slip if the torque tends to rise to a level high enough to damage the machine.

Synchronous belts, sometimes called ***timing belts*** ride on sprockets having mating grooves into which the teeth on the belt seat. This is a positive drive, limited only by the tensile strength of the belt and the shear strength of the teeth. Some cog belts, such are applied to standard V-grooved sheaves. The cogs give the belt greater flexibility and higher efficiency compared with standard belts. They can operate on smaller sheave diameters.

The four principal types of belts are shown, with some of their characteristics, in the following table. *Crowned pulleys* are used for flat belts, and *grooved pulleys,* or *sheaves,* for round and V belts. Timing belts require *toothed wheels,* or *sprockets*. In all cases, the pulley axes must be separated by a certain minimum distance, depending upon the belt type and size, to operate properly.

Table Characteristics of Some Common Belt Types (Figures are cross sections except for the timing belt, which is a side view).

Belt Type	Figure	Joint	Size Range	Center Distance
Flat		Yes	$t = \begin{cases} 0.03 \text{ to } 0.20 \text{ in} \\ 0.75 \text{ to } 5 \text{ mm} \end{cases}$	No upper limit
Round		Yes	$d = \frac{1}{8}$ to $\frac{3}{4}$ in	No upper limit
V		None	$b = \begin{cases} 0.31 \text{ to } 0.91 \text{ in} \\ 8 \text{ to } 19 \text{ mm} \end{cases}$	Limited
Timing		None	$p = 2$ mm and up	Limited

Other characteristics of belts are:

• They may be used for long center distances.

• Except for timing belts, there is some slip and creep, and so the angular-velocity ratio between the driving and driven shafts is neither constant nor exactly equal to the ratio of the pulley diameters.

• In some cases an idler or tension pulley can be used to avoid adjustments in center distance that are ordinarily necessitated by age or the installation of new belts.

The next figure illustrates the geometry of open and closed flat-belt drives. For a flat belt with this drive the belt tension is such that the sag or droop is visible, when the belt is running. Although the top is preferred for the loose side of the belt, for other belt types either the top or the bottom may be used, because their installed tension is usually greater. Two types of reversing drives are possible (see references for details). Reversing can be provided by crossed belt. Crossed belts must be separated to prevent rubbing if high-friction materials are used. Also, reversing can be provided for open-belt drive by adding additional pulleys.

A flat-belt drive with out-of-plane pulleys can be used. The shafts need not be at right angles as in this case. The pulleys must be positioned so that the belt leaves each pulley in the mid-plane of the other pulley face. Other arrangements may require guide pulleys to achieve this condition.

Belt materials: Flat belts are made of urethane and also of rubber-impregnated fabric reinforced with steel wire or nylon cords to take the tension load. One or both surfaces may have a friction surface coating. Flat belts are quiet, they are efficient at high speeds, and they can transmit large amounts of power over long center distances. Usually, flat belting is purchased by the roll and cut and the ends are joined by using special kits furnished by the manufacturer. Two or more flat belts running side by side, instead of a single wide belt, are often used to form a conveying system.

A V belt is made of fabric and cord, usually cotton, rayon, or nylon, and impregnated with rubber. In contrast with flat belts, V belts are used with similar sheaves and at shorter center distances. V belts are slightly less efficient than flat belts, but a number of them can be used on a single sheave, thus making a multiple drive. V belts are made only in certain lengths and have no joints.

Timing belts are made of rubberized fabric and steel wire and have teeth that fit into grooves cut on the periphery of the sprockets. The timing belt does not stretch or slip and consequently transmits power at a constant angular-velocity ratio. The fact that the belt is toothed provides several advantages over ordinary belting. One of these is that no initial tension is necessary, so that fixed-center drives may be used. Another is the elimination of the restriction on speeds; the teeth make it possible to run at nearly any speed, slow or fast. Disadvantages are the first cost of the belt, the necessity of grooving the sprockets, and the attendant dynamic fluctuations caused at the belt-tooth meshing frequency.

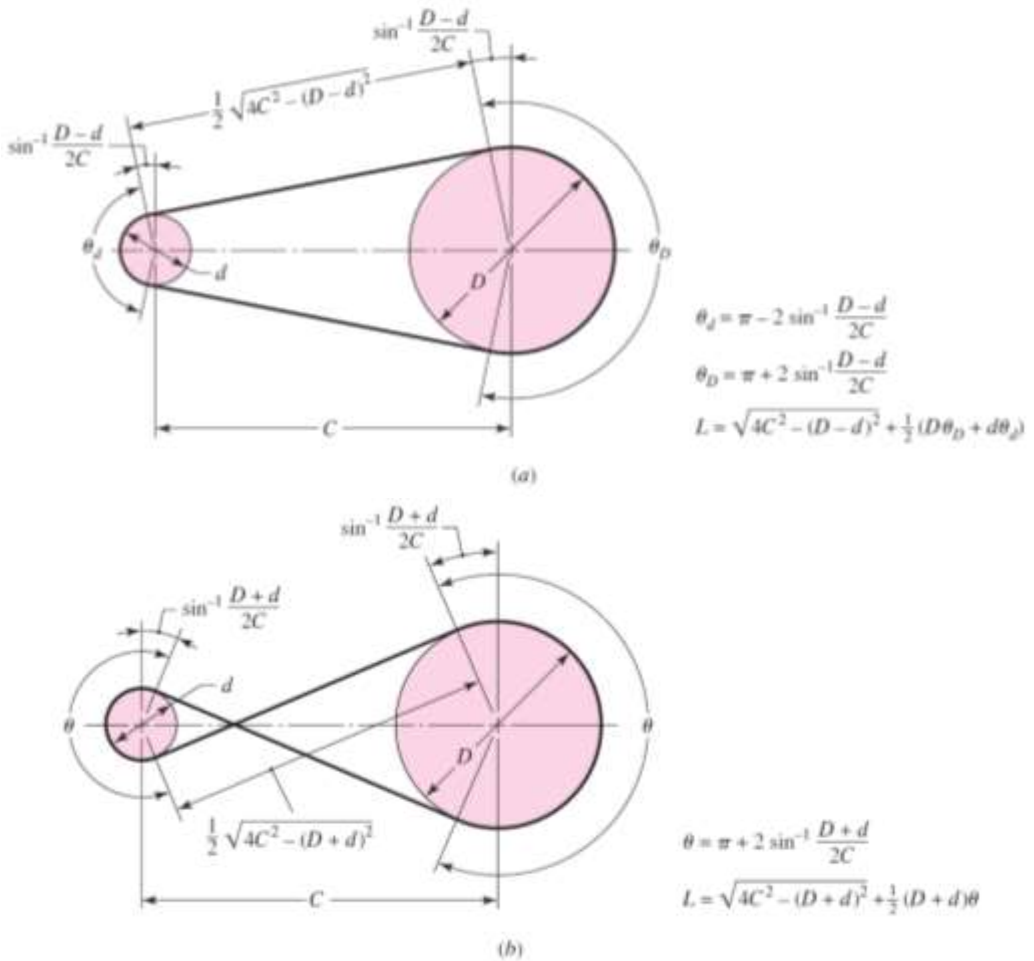


Figure : Flat-belt geometry. (*a*) Open belt. (*b*) Crossed belt.

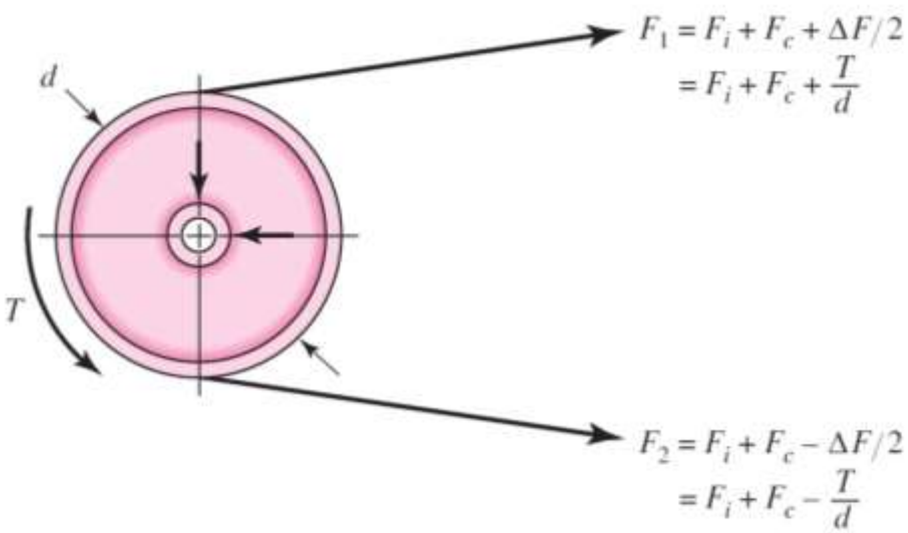


Figure: Forces and torques on a pulley.

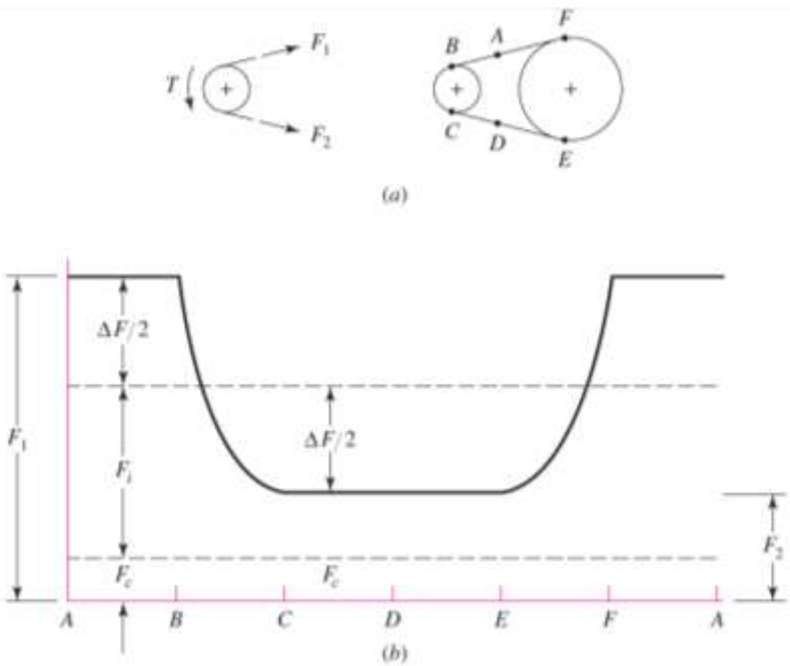


Figure: Flat-belt tensions.

A widely used type of belt, particularly in industrial drives and vehicular applications, is the ***V-belt drive***. The V-shape causes the belt to wedge tightly into the groove, increasing friction and allowing high torques to be transmitted before slipping occurs. Most belts have high-strength cords positioned at the pitch diameter of the belt cross section to increase the tensile strength of the belt. The cords, made from natural fibers, synthetic strands, or steel, are embedded in a firm rubber compound to provide the flexibility needed to allow the belt to pass around the sheave. Often an outer fabric cover is added to give the belt good durability. The selection of commercially available V-belt drives is discussed in the next section. The typical arrangement of the elements of a V-belt drive is shown in the following figure. The important observations to be derived from this arrangement are summarized here:

1. The pulley, with a circumferential groove carrying the belt, is called a *sheave* (usually pronounced "shiv").
2. The size of a sheave is indicated by its pitch diameter, slightly smaller than the outside diameter of the sheave.
3. The speed ratio between the driving and the driven sheaves is inversely proportional to the ratio of the sheave pitch diameters. This follows from the observation that there is no slipping (under normal loads). Thus, the linear speed of the pitch line of both sheaves is the same as and equal to the belt speed, $\mathbf{v_b}$. Then

$$\mathbf{v_b} = R_1\omega_1 = R_2\omega_2$$

Then the angular velocity ratio is

$$R_1 / R_2 = \omega_2 / \omega_1$$

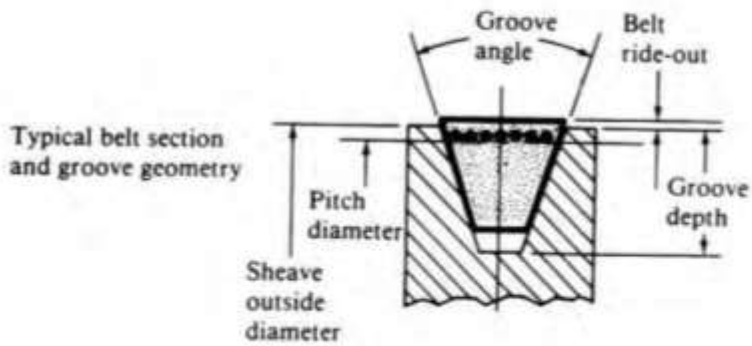


Figure :Cross section of V-belt and sheave groove

4. The relationships between pitch length, *L*. center distance, C, and the sheave diameters are

$$L = 2C + 1.57\,(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C}$$

$$C = \frac{B + \sqrt{B^2 - 32\,(D_2 - D_1)^2}}{16}$$

Where B = 4L – 6.28 (D_2+D_1)

5. The angle of contact of the belt on each sheave is

$$\theta_1 = 180° - 2\,sin^{-1}\left[\frac{D_2 - D_1}{2C}\right]$$

$$\theta_2 = 180° + 2\,sin^{-1}\left[\frac{D_2 - D_1}{2C}\right]$$

These angles are important because commercially available belts are rated with an assumed contact angle of 180^o. This will occur only if the drive ratio is 1 (no speed change). The angle of contact on the smaller of the two sheaves will always be less than 180^o, requiring a lower power rating.

6. The length of the span between the two sheaves, over which the belt is unsupported, is

$$S = \sqrt{C^2 - \left[\frac{D_2 - D_1}{2}\right]^2}$$

This is important for two reasons: You can check the proper belt tension by measuring the amount of force required to deflect the belt at the middle of the span by a given amount. Also, the tendency for the belt to vibrate or whip is dependent on this length.
7. The contributors to the stress in the belt are as follows:
(a) The tensile force in the belt, maximum on the tight side of the belt.
(b) The bending of the belt around the sheaves, maximum as the tight side of the belt bends around the smaller sheave.
(c) Centrifugal forces created as the belt moves around the sheaves.
The maximum total stress occurs where the belt enters the smaller sheave, and the bending stress is a major part. Thus, there are recommended minimum sheave diameters for standard belts. Using smaller sheaves drastically reduces belt life.
8. The design value of the ratio of the tight side tension to the slack side tension is 5.0 for V-belt drives. The actual value may range as high as 10.0.

Standard Belt Cross Sections
Commercially available belts are made to one of the different standards. The alignment between the inch sizes and the metric sizes indicates that the paired sizes are actually the same cross section. A "soft conversion" was used to rename the familiar inch sizes with the number for the metric sizes giving the nominal top width in millimeters. The nominal value of the included angle between the sides of the V-groove ranges from 30° to 42°. The angle on the belt may be slightly different to achieve a tight fit in the groove. Some belts are designed to "ride out" of the groove somewhat. One example of these standards that used in Automotive applications is shown below.

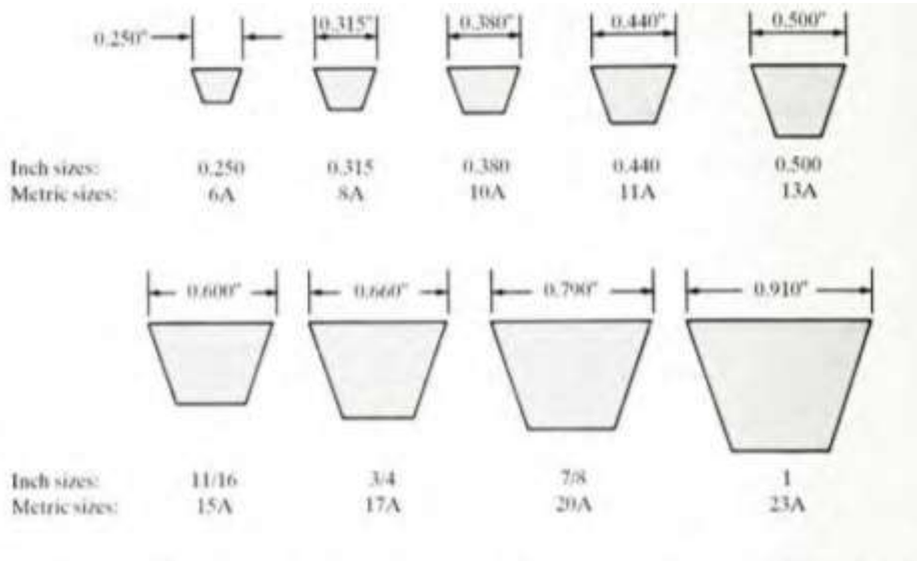


Figure: Automotive V-belts

V-belt Design:
The factors involved in selection of a V-belt and the driving and driven sheaves and proper installation of the drive are summarized in this section. Abbreviated examples of the data available from suppliers are given for illustration. Catalogs contain extensive data, and step-by-step instructions are given for their use. The basic data required for drive selection are the following:
• The rated power of the driving motor or other prime mover
• The service factor based on the type of driver and driven load
• The center distance
• The power rating for one belt as a function of the size and speed of the smaller sheave
• The belt length
• The size of the driving and driven sheaves
• The correction factor for belt length
• The correction factor for the angle of wrap on the smaller sheave
• The number of belts
• The initial tension on the belt
Many design decisions depend on the application and on space limitations. A few guidelines are given here:
• Adjustment for the center distance must be provided in both directions from the nominal value. The center distance must be shortened at the time of installation to enable the belt to be placed in the grooves of the sheaves without force. Provision for increasing the center distance must be made to permit the initial tensioning of the drive and to take up for belt stretch. Manufacturers' catalogs give the data. One convenient way to accomplish the adjustment is the use of a take-up unit.
• If fixed centers are required, idler pulleys should be used. It is best to use a grooved idler on the inside of the belt, close to the large sheave. Adjustable tensioners are commercially available to carry the idler.
• The nominal range of center distances should be

$$D_2 < C < 3\,(D_2 + D_1)$$

• The angle of wrap on the smaller sheave should be greater than 120°.
• Most commercially available sheaves are cast iron, which should be limited to 30-m/sec. belt speed.
• Consider an alternative type of drive, such as a gear type or chain, if the belt speed is less than 5 m/sec.
• Avoid elevated temperatures around belts.
• Ensure that the shafts carrying mating sheaves are parallel and that the sheaves are in alignment so that the belts track smoothly into the grooves.
• In multi-belt installations, matched belts are required. Match numbers are printed on industrial belts, with 50 indicating a belt length very close to nominal. Longer belts carry match numbers above 50; shorter belts below 50.
• Belts must be installed with the initial tension recommended by the manufacturer. Tension should be checked after the first few hours of operation because seating and initial stretch occur.

Design Data
Catalogs typically give several dozen pages of design data for the various sizes of belts and sheave combinations to ease the job of drive design. The data typically are given in tabular form (.see References). Graphical form is also used so that you can get a feel for the variation in performance with design choices.

Note that the power used is *design power,* the rated power of the prime mover times the service factor from Tables.

Figures 7-10, 7-11, and 7-12 give the rated power per belt for the three cross secfions as a function of the pitch diameter of the smaller sheave and its speed of rotation. The i„u..i.,^ vertical lines in each figure give the standard sheave pitch diameters available.

TABLE : V-belt service factors

	Driver type					
	AC motors: Normal torque[a] DC motors: Shunt-wound Engines: Multiple-cylinder			AC motors: High torque[b] DC motors: Series-wound, compound-wound Engines: 4-cylinder or less		
Driven machine type	<6h per day	6-15h per day	>l5h per day	<6h per day	6-15 h per day	>15h per day
Agitators, blowers, fans, centrifugal pumps, light conveyors	1.0	1.1	1.2	1.1	1.2	1.3
Generators, machine tools, mixers, gravel conveyors	1.1	1.2	1.3	1.2	1.3	1.4
Bucket elevators, textile machines, hammer mills, heavy conveyors	1.2	1.3	1.4	1.4	1.5	1.6
Crushers, ball mills, hoists, rubber extruders	1.3	1.4	1.5	1.5	1.6	1.8
Any machine that can choke	2.0	2.0	2.0	2.0	2.0	2.0

[a]. Synchronous, split-phase, three-phase with starting torque or breakdown torque less than 175% of full-load torque.

[b]. Single-phase, three-phase with starting torque or breakdown torque greater than 175% of full-load torque.

Synchronous Belt Drives

Synchronous belts are constructed with ribs or teeth across the underside of the belt, as shown in following figure. The teeth mate with corresponding grooves in the driving and driven pulleys, called *sprockets,* providing a positive drive without slippage. Therefore, there is a fixed relationship between the speed of the driver and the speed of the driven sprocket. For this reason synchronous belts are often called *timing belts.* In contrast, V-belts can creep or slip with respect to their mating sheaves, especially under heavy loads and varying power demand. Synchronous action is critical to the successful operation of such systems as printing, material handling, packaging, and assembly. Synchronous belt drives are increasingly being considered for applications in which gear drives or chain drives had been used previously. Figure shows a synchronous belt mating with the toothed driving sprocket. Typical driving and driven sprockets are shown in Figure. At least one of the two sprockets will have side flanges to ensure that the belt does not move axially. Figure shows the four common tooth pitches and sizes for commercially available synchronous belts. The pitch is the distance from the center of one tooth to the center of the next adjacent tooth. Standard pitches are 5 mm, 8 mm, 14 mm, and 20 mm. Figure 7-3(c) shows detail of the construction of the cross section of a synchronous belt. The tensile strength is provided predominantly by high-strength cords made from fiberglass or similar materials. The cords are encased in a flexible rubber backing material, and the teeth are formed integrally with the backing. Often a fabric covering is used on those parts ofthe belt that contact the sprockets to provide additional wear resistance and higher net shear strength for the teeth. Various widths ofthe belts are available for each given pitch to provide a wide range of power transmission capacity. Commercially available sprockets typically employ .split-taper bushings in their hubs with a precise bore that provides a clearance of only 0.001 to 0.002 in (0.025 to 0.050 mm) relative to the shaft diameter on which it is to be mounted. Smooth, balanced, concentric operation results. The process of .selecting appropriate components for a synchronous belt drive is similar to that already discussed for V-belt drives.

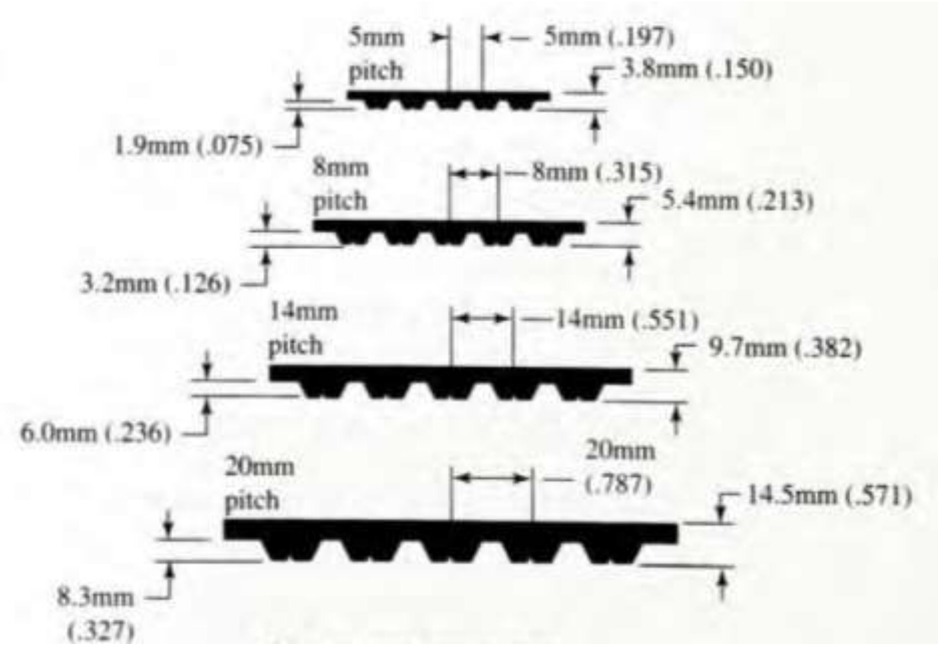


Figure: Dimensions of standard synchronous belts (Numbers in parentheses are inches)

Manufacturers provide selection guides similar to those shown in Figure 7-19 that give the relationship between design power and the rotational speed of the smaller sprocket. These are used to determine the basic belt pitch required.
Also, numerous pages of performance data are given showing the power transmission capacity for many combinations of belt width, driving and driven sprocket size, and center distances between the axes of the sprockets for specific belt lengths. In general the selection process involves the following steps. Refer to data and design procedures for specific manufacturers as listed in Internet sites 2-5.

General Selection Procedure for Synchronous Belt Drives

1. Specify the speed of the driving sprocket (typically a motor or engine) and the desired
speed of the driven sprocket.
2. Specify the rated power for the driving motor or engine.
3. Determine a service factor, using manufacturers' recommendations, considering the type of driver and the nature of the driven machine.
4. Calculate the design power by multiplying the driver rated power by the service factor.
5. Determine the required pitch of the belt from a specific manufacturer's data.
6. Calculate the speed ratio between the driver and the driven sprocket.
7. Select several candidate combinations of the number of teeth in the driver sprocket to that in the driven sprocket that will produce the desired ratio.
8. Using the desired range of acceptable center distances, determine a standard belt length that will produce a suitable value.

9. A belt-length correction factor may be required. Catalog data will show factors less than 1.0 for shorter center distances and greater than 1.0 for longer center distances. This reflects the frequency with which a given part of the belt encounters the high-stress area as it enters the smaller sprocket. Apply the factor to the rated power capacity for the belt.
10. Specify the final design details for the sprockets such as flanges, type and size of bushings in the hub, and the bore size to match the mating shafts.
11. Summarize the design, check compatibility with other components of the system, and prepare purchasing documents. Installation of the sprockets and the belt requires a nominal amount of center distance allowance to enable the belt teeth to slide into the sprocket grooves without force. Subsequently, the center distance will normally have to be adjusted outward to provide a suitable amount of initial tension as defined by the manufacturer. The initial tension is typically less than that required for a V-belt drive. Idlers can be used to take up slack if fixed centers are required between the driver and driven sprockets. However, they may decrease the life of the belt. Consult the manufacturer.

In operation, the final tension in the tight side of the belt is much less than that developed by a V-belt and the slack side tension is virtually zero. The results are lower net forces in the belt, lower side loads on the shafts carrying the sprockets, and reduced bearing loads.

Chain Drives

Basic features of chain drives include a constant ratio, since no slippage or creep is involved; long life; and the ability to drive a number of shafts from a single source of power.

Roller chains have been standardized as to sizes by the ANSI. The figure shows the nomenclature. The pitch is the linear distance between the centers of the rollers. The width is the space between the inner link plates. These chains are manufactured in single, double, triple, and quadruple strands. The dimensions of standard sizes are listed in following table.

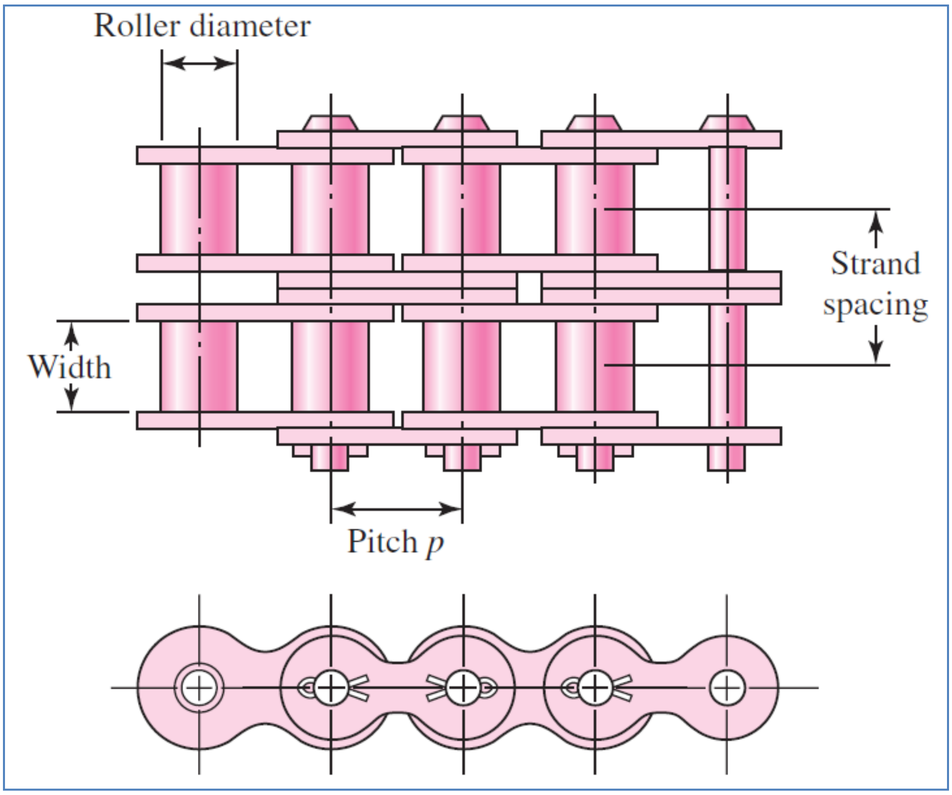


Figure : Portion of a double-strand roller chain.

Table : Dimensions of American Standard Roller Chains—Single Strand *Source:* Compiled from ANSI B29.1-1975.

ANSI Chain Number	Pitch in (mm)	Width in (mm)	Minimum Tensile, Strength lbf (N)	Average Weight, lbf/ft (N/m)	Roller Diameter in (mm)	Multiple-Strand Spacing in (mm)
25	0.250	0.125	780	0.09	0.130	0.252
	(6.35)	(3.18)	(3470)	(1.31)	(3.30)	(6.40)
35	0.375	0.188	1760	0.21	0.200	0.399
	(9.52)	(4.76)	(7830)	(3.06)	(5.08)	(10.13)
41	0.500	0.25	1500	0.25	0.306	—
	(12.70)	(6.35)	(6670)	(3.65)	(7.77)	—
40	0.500	0.312	3130	0.42	0.312	0.566
	(12.70)	(7.94)	(13920)	(6.13)	(7.92)	(14.38)
50	0.625	0.375	4880	0.69	0.400	0.713
	(15.88)	(9.52)	(21700)	(10.1)	(10.16)	(18.11)
60	0.750	0.500	7030	1.00	0.469	0.897
	(19.05)	(12.7)	(31300)	(14.6)	(11.91)	(22.78)
80	1.000	0.625	12500	1.71	0.625	1.153
	(25.40)	(15.88)	(55600)	(25.0)	(15.87)	(29.29)
100	1.250	0.750	19500	2.58	0.750	1.409
	(31.75)	(19.05)	(86700)	(37.7)	(19.05)	(35.76)
120	1.500	1.000	28000	3.87	0.875	1.789
	(38.10)	(25.40)	(124500)	(56.5)	(22.22)	(45.44)
140	1.750	1.000	38000	4.95	1.000	1.924
	(44.45)	(25.40)	(169000)	(72.2)	(25.40)	(48.87)
160	2.000	1.250	50000	6.61	1.125	2.305
	(50.80)	(31.75)	(222000)	(96.5)	(28.57)	(58.55)
180	2.250	1.406	63000	9.06	1.406	2.592
	(57.15)	(35.71)	(280000)	(132.2)	(35.71)	(65.84)
200	2.500	1.500	78000	10.96	1.562	2.817
	(63.50)	(38.10)	(347000)	(159.9)	(39.67)	(71.55)
240	3.00	1.875	112000	16.4	1.875	3.458
	(76.70)	(47.63)	(498000)	(239)	(47.62)	(87.83)

Table : Roller chain sizes

Chain number	Pitch (in)	Roller diameter	Roller width	Link plate thickness	Average tensile strength (lb)
25	*1/4*	None	-	0.030	925
35	*3/8*	None	-	0.050	2100
41	*1/2*	0.306	0.250	0.050	2000
40	*1/2*	0.312	0.312	0.060	3700
50	*5/8*	0.400	0.375	0.080	6100
60	*3/4*	0.469	0.500	0.094	8500
80	1	0.626	0.625	0.125	14 500
100	1 *1/4*	0.750	0.750	0.156	24 000
120	1 *1/2*	0.875	1.000	0.187	34 000
140	1 *3/4*	1.000	1.000	0.219	46 000
160	2	1.125	1.250	0.250	58 000
180	2 *1/4*	1.406	1.406	0.281	80 000
200	2 *1/2*	1.562	1.500	0.312	95 000
240	3	1.875	1.875	0.375	130 000

The following figure shows a sprocket driving a chain and rotating in a counterclockwise direction. Denoting the chain pitch by p, the pitch angle by γ , and the pitch diameter of the sprocket by D, from the trigonometry of the figure we see

$$\sin\frac{\gamma}{2} = \frac{p/2}{D/2} \quad or \quad D = \frac{p}{\sin(\gamma/2)}$$

Since $\gamma = 360^\circ/N$, where N is the number of sprocket teeth, the above equation can be written

$$D = \frac{p}{\sin(180°/N)}$$

The angle $\gamma/2$, through which the link swings as it enters contact, is called the *angle of articulation*. It can be seen that the magnitude of this angle is a function of the number of teeth. Rotation of the link through this angle causes impact between the rollers and the sprocket teeth and also wear in the chain joint. Since the life of a properly selected drive is a function of the wear and the surface fatigue strength of the rollers, it is important to reduce the angle of articulation as much as possible. The number of sprocket teeth also affects the velocity ratio during the rotation through the pitch angle γ . At the position shown in the figure, the chain AB is tangent to the pitch circle of the sprocket. However, when the sprocket has turned an angle of $\gamma/2$, the chain line AB moves closer to the center of rotation of the sprocket.

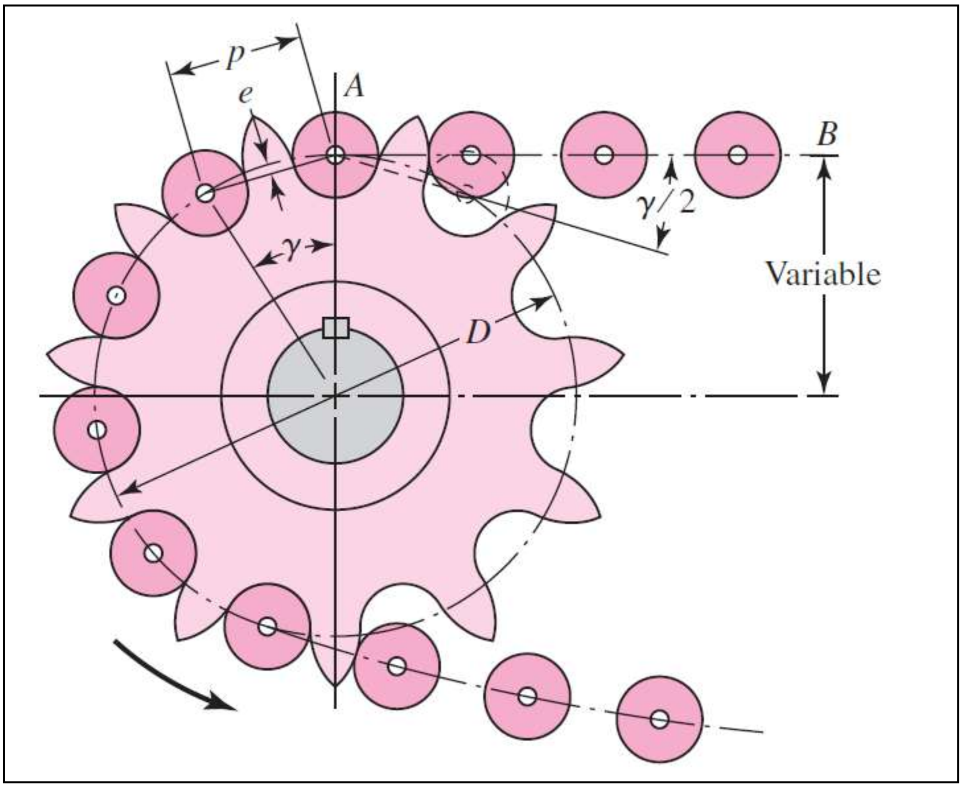


Figure: Engagement of a chain and sprocket.

This means that the chain line *AB* is moving up and down, and that the lever arm varies with rotation through the pitch angle, all resulting in an uneven chain exit velocity. You can think of the sprocket as a polygon in which the exit velocity of the chain depends upon whether the exit is from a corner, or from a flat of the polygon. Of course, the same effect occurs when the chain first enters into engagement with the sprocket. The chain velocity *V* is defined as the number of feet coming off the sprocket per unit time. Thus the chain velocity in feet per minute is

$$V = Npn/12$$

where N = number of sprocket teeth
p = chain pitch, in
n = sprocket speed, rev/min

The maximum exit velocity of the chain is

$$v_{max} = \frac{\pi D n}{12} = \frac{\pi n p}{12\sin(\gamma/2)}$$

where the first Equation has been substituted for the pitch diameter *D*. The minimum exit velocity occurs at a diameter *d*, smaller than *D*. Using the geometry of the figure, we find

$$d = D\cos(\gamma/2)$$

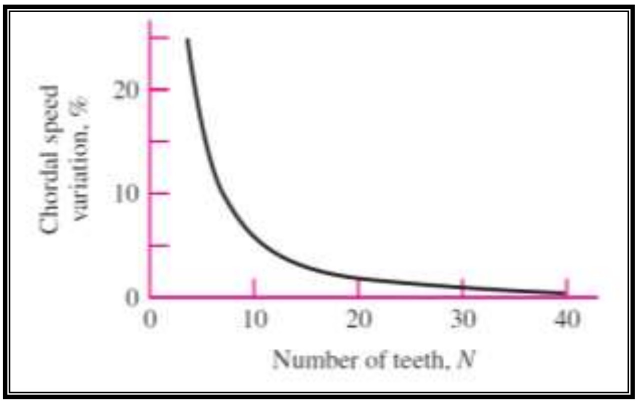
Thus the minimum exit velocity is

$$v_{min} = \frac{\pi d n}{12} = \frac{\pi n p \cos(\gamma/2)}{12\sin(\gamma/2)}$$

Now substituting $\gamma/2 = 180^\circ/N$ and employing the previous equations, we find the speed variation to be

$$\frac{\Delta V}{V} = \frac{v_{max} - v_{min}}{V} = \frac{\pi}{N}\left[\frac{1}{\sin(180°/N)} - \frac{1}{\tan(180°/N)}\right]$$

This is called the *chordal speed variation* and is plotted in the following figure. When chain drives are used to synchronize precision components or processes, due consideration must be given to these variations. For example, if a chain drive synchronized the cutting of photographic film with the forward drive of the film, the lengths of the cut sheets of film might vary too much because of this chordal speed variation. Such variations can also cause vibrations within the system.



Although a large number of teeth is considered desirable for the driving sprocket, in the usual case it is advantageous to obtain as small a sprocket as possible, and this requires one with a small number of teeth. For smooth operation at moderate and high speeds it is considered good practice to use a driving sprocket with at least 17 teeth; 19 or 21 will, of course, give a better life expectancy with less chain noise. Where space limitations are severe or for very slow speeds, smaller tooth numbers may be used by sacrificing the life expectancy of the chain.

Driven sprockets are not made in standard sizes over 120 teeth, because the pitch elongation will eventually cause the chain to "ride" high long before the chain is worn out. The most successful drives have velocity ratios up to 6:1, but higher ratios may be used at the sacrifice of chain life.

Roller chains seldom fail because they lack tensile strength; they more often fail because they have been subjected to a great many hours of service. Actual failure may be due either to wear of the rollers on the pins or to fatigue of the surfaces of the rollers.

Roller-chain manufacturers have compiled tables that give the horsepower capacity corresponding to a life expectancy of 15 kh for various sprocket speeds. These capacities are tabulated in Table 17–20 for 17-tooth sprockets. Table 17–21 displays available tooth counts on sprockets of one supplier. Table 17–22 lists the tooth correction factors for other than 17 teeth. Table 17–23 shows the multiple-strand factors K_2. The capacities of chains are based on the following:

- 15 000 h at full load
- Single strand
- ANSI proportions
- Service factor of unity
- 100 pitches in length
- Recommended lubrication
- Elongation maximum of 3 percent
- Horizontal shafts
- Two 17-tooth sprockets

The fatigue strength of link plates governs capacity at lower speeds. The American Chain Association (ACA) publication *Chains for Power Transmission and Materials Handling* (1982) gives, for single-strand chain, the nominal power H_1, link-plate limited, as $H_1 = 0.004N_1^{1.08}$

$$H_1 = 0.004\, N_1^{1.08}\, n_1^{0.9} p^{(3-0.07\, p)} \qquad [hp]$$

and the nominal power H_2, roller-limited, as

$$H_2 = \frac{1000\, K_r\, N_1^{1.5}\, p^{0.8}}{n_1^{1.5}} \qquad [hp]$$

where N_1 = number of teeth in the smaller sprocket

n_1 = sprocket speed, rev/min

p = pitch of the chain, in

K_r = 29 for chain numbers 25, 35; 3.4 for chain 41; and 17 for chains 40–240

The constant 0.004 becomes 0.0022 for no. 41 lightweight chain. The nominal horsepower in the Table is $H_{nom} = \min(H_1, H_2)$. For example, for $N_1 = 17$, $n_1 = 1000$ rev/min, no. 40 chain with $p = 0.5$ in, from equations,

$H_1 = 0.004(17)^{1.08} 1000^{0.9} 0.5^{[3-0.07(0.5)]} = 5.48$ hp

And,

$H_2 = 1000(17)17^{1.5}(0.5^{0.8})/1000^{1.5} = 21.64$ hp

The tabulated value in the table 17–20 is $H_{tab} = \min(5.48, 21.64) = 5.48$ hp.

It is preferable to have an odd number of teeth on the driving sprocket (17, 19, . . .) and an even number of pitches in the chain to avoid a special link. The approximate length of the chain L in pitches is

$$\frac{L}{p} = \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2\, C/p}$$

The center-to-center distance C is given by

$$C = \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\,\right]$$

Where

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p}$$

The allowable power H_a is given by

$$H_a = K_1 K_2 H_{tab}$$

where K_1 = correction factor for tooth number other than 17 (see the above table)

K_2 = strand correction (see the above table)

The horsepower that must be transmitted H_d is given by

$$H_d = H_{nom}\, K_s\, n_d$$

The equation that find H_1 above is the basis of the pre-extreme power entries (vertical entries) of the Table, and the chain power is limited by link-plate fatigue. The equation that find H_2 is the basis for the post-extreme power entries of these tables, and the chain power performance is limited by impact fatigue. The entries are for chains of 100 pitch length and 17-tooth sprocket. For a deviation from this

$$H_2 = 1000\left[K_r \left(\frac{N_1}{n_1}\right)^{1.5} p^{0.8} \left(\frac{L_p}{100}\right)^{0.4} \left(\frac{15\,000}{h}\right)^{0.4}\right] \qquad [hp]$$

where L_p is the chain length in pitches and h is the chain life in hours. Viewed from a deviation viewpoint, this equation can be written as a trade-off equation in the following form:

$$\frac{H_2^{2.5}\, h}{N_1^{3,75} L_p} = Constant$$

If tooth-correction factor K_1 is used, then omit the term $N_1^{3.75}$.

Note that $(N_1^{1.5})^{2.5} = N_1^{3.75}$

In the above equation one would expect the h/L_p term because doubling the hours can require doubling the chain length, other conditions constant, for the same number of cycles. Our experience with contact stresses leads us to expect a load (tension) life relation of the form F^aL = constant. In the more complex circumstance of roller-bushing impact, the Diamond Chain Company has identified a = 2.5.

The maximum speed (rev/min) for a chain drive is limited by galling between the pin and the bushing. Tests suggest

$n_1 \leq 1000[\, 82.5 / (7.95p\, (1.0278)^{N_1}\, (1.323)^{F/1000}]^{\,1/(1.59 \log p+1.873)}$ rev/min

where *F* is the chain tension in pounds.

Table: Rated Horsepower Capacity of Single-Strand Single-Pitch Roller Chain for a 17-Tooth Sprocket *Source:* Compiled from ANSI B29.1-1975 information only section, and from B29.9-1958.

Sprocket Speed, rev/min	ANSI Chain Number					
	25	35	40	41	50	60
50	0.05	0.16	0.37	0.20	0.72	1.24
100	0.09	0.29	0.69	0.38	1.34	2.31
150	0.13*	0.41*	0.99*	0.55*	1.92*	3.32
200	0.16*	0.54*	1.29	0.71	2.50	4.30
300	0.23	0.78	1.85	1.02	3.61	6.20
400	0.30*	1.01*	2.40	1.32	4.67	8.03
500	0.37	1.24	2.93	1.61	5.71	9.81
600	0.44*	1.46*	3.45*	1.90*	6.72*	11.6
700	0.50	1.68	3.97	2.18	7.73	13.3
800	0.56*	1.89*	4.48*	2.46*	8.71*	15.0
900	0.62	2.10	4.98	2.74	9.69	16.7
1000	0.68*	2.31*	5.48	3.01	10.7	18.3
1200	0.81	2.73	6.45	3.29	12.6	21.6
1400	0.93*	3.13*	7.41	2.61	14.4	18.1
1600	1.05*	3.53*	8.36	2.14	12.8	14.8
1800	1.16	3.93	8.96	1.79	10.7	12.4
2000	1.27*	4.32*	7.72*	1.52*	9.23*	10.6
2500	1.56	5.28	5.51*	1.10*	6.58*	7.57
3000	1.84	5.64	4.17	0.83	4.98	5.76
Type A	**Type B**				**Type C**	

*Estimated from ANSI tables by linear interpolation.
Note: Type A—manual or drip lubrication; type B—bath or disk lubrication; type C—oil-stream lubrication.

Table: Rated Horsepower Capacity of Single-Strand Single-Pitch Roller Chain for a 17-Tooth Sprocket *Source:* Compiled from ANSI B29.1-1975 information only section, and from B29.9-1958 (*Continued*)

Sprocket Speed, rev/min	**Lubrication Type**	**ANSI Chain Number 80**	**100**	**120**	**140**	**160**	**180**	**200**	**240**
50	**Type A**	2.88	5.52	9.33	14.4	20.9	28.9	38.4	61.8
100	**Type A**	5.38	10.3	17.4	26.9	39.1	54.0	71.6	115
150	**Type B**	7.75	14.8	25.1	38.8	56.3	77.7	103	166
200	**Type B**	10.0	19.2	32.5	50.3	72.9	101	134	215
300	**Type B**	14.5	27.7	46.8	72.4	105	145	193	310
400	**Type B**	18.7	35.9	60.6	93.8	136	188	249	359
500	**Type B**	22.9	43.9	74.1	115	166	204	222	0
600	**Type B**	27.0	51.7	87.3	127	141	155	169	
700	**Type B**	31.0	59.4	89.0	101	112	123	0	
800	**Type B**	35.0	63.0	72.8	82.4	91.7	101		
900	**Type B**	39.9	52.8	61.0	69.1	76.8	84.4		
1000	**Type B**	37.7	45.0	52.1	59.0	65.6	72.1		
1200	**Type C**	28.7	34.3	39.6	44.9	49.9	0		
1400	**Type C**	22.7	27.2	31.5	35.6	0			
1600	**Type C**	18.6	22.3	25.8	0				
1800	**Type C**	15.6	18.7	21.6					
2000	**Type C**	13.3	15.9	0					
2500	**Type C**	9.56	0.40						
3000	**Type C'**	7.25	0						

Note: Type A—manual or drip lubrication; type B—bath or disk lubrication; type C—oil-stream lubrication; type C'—type C, but this is a galling region; submit design to manufacturer for evaluation.

Table : Single-Strand Sprocket Tooth Counts Available from One Supplier*

No.	Available Sprocket Tooth Counts
25	8-30, 32, 34, 35, 36, 40, 42, 45, 48, 54, 60, 64, 65, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
35	4-45, 48, 52, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
41	6-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
40	8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
50	8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
60	8-60, 62, 63, 64, 65, 66, 67, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
80	8-60, 64, 65, 68, 70, 72, 76, 78, 80, 84, 90, 95, 96, 102, 112, 120
100	8-60, 64, 65, 67, 68, 70, 72, 74, 76, 80, 84, 90, 95, 96, 102, 112, 120
120	9-45, 46, 48, 50, 52, 54, 55, 57, 60, 64, 65, 67, 68, 70, 72, 76, 80, 84, 90, 96, 102, 112, 120
140	9-28, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 42, 43, 45, 48, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 96
160	8-30, 32–36, 38, 40, 45, 46, 50, 52, 53, 54, 56, 57, 60, 62, 63, 64, 65, 66, 68, 70, 72, 73, 80, 84, 96
180	13-25, 28, 35, 39, 40, 45, 54, 60
200	9-30, 32, 33, 35, 36, 39, 40, 42, 44, 45, 48, 50, 51, 54, 56, 58, 59, 60, 63, 64, 65, 68, 70, 72
240	9-30, 32, 35, 36, 40, 44, 45, 48, 52, 54, 60

*Morse Chain Company, Ithaca, NY, Type B hub sprockets.

Table : Tooth Correction Factors, *K*1

Number of Teeth on Driving Sprocket	*K*1 Pre-extreme Horsepower	*K*1 Post-extreme Horsepower
11	0.62	0.52
12	0.69	0.59
13	0.75	0.67
14	0.81	0.75
15	0.87	0.83
16	0.94	0.91
17	1.00	1.00
18	1.06	1.09
19	1.13	1.18
20	1.19	1.28
N	(*N*1/17)1.08	(*N*1/17)1.5

Table : Multiple-Strand Factors, K_2

Number of Strands	$\boldsymbol{K_2}$
1	1.0
2	1.7
3	2.5
4	3.3
5	3.9
6	4.6
8	6.0

Lubrication of roller chains is essential in order to obtain a long and trouble-free life. Either a drip feed or a shallow bath in the lubricant is satisfactory. A medium or light mineral oil, without additives, should be used. Except for unusual conditions, heavy oils and greases are not recommended, because they are too viscous to enter the small clearances in the chain parts.

Gears

Introduction:

In transmitting rotary power from one shaft to another, gears provide a positive ratio type drive. If the shafts are parallel any one of three type may be used, spur, helical, or herring. Spiral gears are used to connect two shafts and which are non-intersection. Worm and worm gear are used where high speed ratios where the shafts are non-intersecting and at right angles. Bevel gears are often used where two shaft are at right angle to each other. Spiral bevel may be used in the same type application as straight-tooth bevel gear but are capable of high speed and quitter operation. Hypoid are similar to spiral bevel gear except that the extension of the center lines are non-intersecting. Hypoid gear are originally developed for the automotive rear-axle drive. Rack and pinion drive are used are used when it is desired to transfer the rotary motion of the part into translating motion for the other part and vice versa.

Gear classification:

Gears are classified according to:

- Position of shafts:
  - Parallel (spur)
  - Intersecting (bevel)
- Position of teeth with respect to gear axis:
  - Straight tooth
  - Helical
  - Curved teeth
- Atmospheric conditions:
  - Open
  - Closed
- Manufacturing accuracy (12 degree of accuracy for spur gear):
  - Fine surface finish.
  - Coarse surface finish.

Profile of teeth:

- Involute
- Cycloid

Material:

- Steel
- Cast iron.
- Bronzes.
- Non-metallic materials

Types of gears

- Spur gear:
- Helical gear
- Herringbone (double helical
- Bevel gear
- Worm and Worm gear: irreversible
- Rack and pinion

Advantages of gears:

- High efficiency (except the worm and worm gear)
- So compact

Disadvantages of gears:

High cost

Gear forces:

Spur gear force components are:

Tangential force:

$$\mathbf{F_t = T / r}$$

where T = gear torque and r = pitch radius of the gear

Radial force (always toward the center of the gear)

$$\mathbf{F_r = F_t \tan \phi}$$

where ϕ is the pressure angle.

Helical gear force depends upon how the pressure angle defined. There are two standards:

(1) The pressure angle ϕ is measured in the plane perpendicular to the axis of the gear. The components are:

(One) Tangential force

$$\mathbf{F_t = T / r}$$

(Two) Radial force

$$\mathbf{F_r = F_t \tan \phi.}$$

(Three) Axial (thrust) force

$$\mathbf{F_a = F_t \tan \alpha}$$

where α is helix angle measured from the axis of the gear.

(2) The pressure angle ϕ is measured in the plane normal to a tooth. The components are:

(One) Tangential force

$$\mathbf{F_t = T / r}$$

(Two) Radial force

$$\mathbf{F_r = F_t (\tan \phi_n / \cos \alpha)}$$

(Three) Axial (thrust) force

$$\mathbf{F_a = F_t \tan \alpha}$$

where α is helix angle measured from the axis of the gear.

Straight tooth bevel gear force component are:

Tangential force

$$\mathbf{F_t = T_t / r}$$

where this force is acting at the mean pitch diameter.

Radial force

$$\mathbf{F_r = F_t \tan \phi}$$

where ϕ is the pressure angle. This force can be resolved into two component; $\mathbf{F_p}$

$$\mathbf{(F_p = F_t \tan \phi \sin \beta)}$$

along the shaft axis of the pinion and $\mathbf{F_g}$

$$\mathbf{(F_g = F_t \tan \phi \cos \beta)}$$

along the shaft axis of the gear.

For other type of gears see any one of the references.

Nomenclature of Spur gear

Pitch diameter (d_o):

It is an imagining circle which if assuming rolling action, would give the same motion as actual gear. It very importat value in speed calculation.

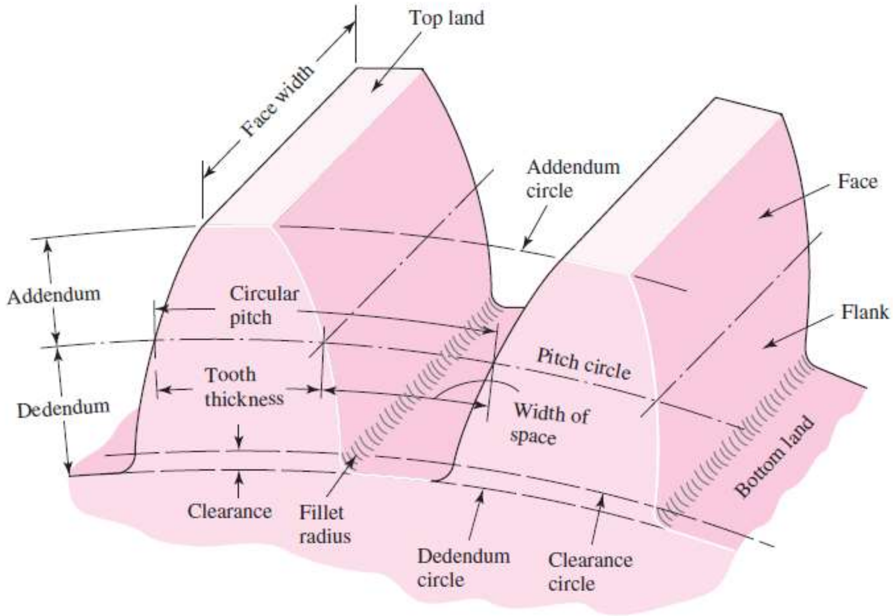


Figure : Nomenclature of spur-gear teeth.

Addendum (h_k):

It is a radial distance of tooth from the pitch to the top of the tooth.

Dedendum (h_f):

It is a radial distance of tooth from the pitch to the bottom of the tooth.

Addendum circle (d_k):

It is the circle drawn through the top of the teeth

$$d_k = d_o + 2\ h_k$$

Dedendum circle (d_f)

It is the circle drawn through the bottom of the teeth

$$d_f = d_o - 2\ h_f$$

Circular pitch (t_o):

It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. Thus the circular pitch is equal to the sum of the tooth thickness and the width of space.

$$t_o = \pi\ d_o / z = \pi\ m$$

where z is the number of teeth and m is the module.

Module (m):

It is the ratio of the pitch circle diameter in (mm) to the number of teeth. They customary unit of length used is the millimeter. The module is the represented of tooth size in SI.

$$m = d_o / z = t_o / \pi$$

The diameteral pitch P is the ratio of the number of teeth on the gear to the pitch diameter. Thus, it is the reciprocal of the module.

Whole depth (h):

It is a radial distance between the Addendum: Dedendum

$$h = h_k + h_f:$$

Clearance (S_k):

It is the radial distance from the top of the tooth to the bottom of the tooth in meshing gear. The circle passing through the top of the meshing gear is known as a clearance circle

Center distance (a):

$$a = (d_{o1} + d_{o2}) /2 = S_k + d_{f1}/2 + d_{k2}/2$$

Tooth thickness (S_o):
It is the width of the tooth measured along the pitch circle.

Tooth space (L_o):
It is the width of the space between two adjacent teeth measured along the pitch circle.

Back lash (S_d) :
It is the difference between the tooth space and tooth thickness as measured on the pitch circle. It is the amount by which the width of tooth space exceeds the thickness of the engaging tooth measured on the pitch circles.

Face width (b):
It is the width of the gear tooth measured parallel to its axis.

Profile:
It is the curve formed by the face and flank of the tooth.

Path of contact:
It is the bath traced by the point of contact of two teeth from the beginning to the end of engagement.

Pressure angle (α):
It is the angle between the common normal to the two teeth in contact and common tangent to the pitch circle. Standard value are 14.4°, 20° and 25°

$$\textbf{Base circle diameter = Pitch diameter * cos. } \alpha$$

where the base circle is the circle which the tooth profile start with.

Conjugate action:

Mating gear teeth acting against each other to produce rotary motion are similar to cams. When the tooth profiles or cams are designed so as to produce a constant angular-velocity during meshing, they said to have conjugate action.

Form of gear tooth profile:

Since the velocity ratio of two gears is required to be constant, than the tooth profile must satisfy the fundamental requirements of a pair of curves in direct sliding contact. The most common forms are the involute and cycloid.

Involute:

The involute is the locus of a point as a string which is unwounded from a circle, the circle is known as base circle. The string is kept tight during unwinding process.
One way to construct this form is to divide part of the circumference of the base circle by draw equal angles (θ) from the circle center. The accuracy of the curve depends in

the value of this angle(the smaller the value the better the result). The next step is to number the cross points between the radii and the circumference by 0, 1, 2, ..n. Draw a tangent from each point on the circumference (this is with right angle with the radius at this point). The profile starts at point 0. The second point will be at a distance of (r θ) from point 1 measured in the tangent at this point. The third point will be at a distance of (2 r θ) from point 2 measured in the tangent at this point. The following points are drawn at a distance of (n r θ) from the point n measured in the tangent of that point.

Low of gearing:

The angular velocities are universally proportional to the parts in which the line of center is divided by the common normal at the point of contact. Therefore for constant angular velocity ratio the common normal through the point of contact must divide the line of centers in a fixed ratio. This is the low of gearing.

$$\omega_1 / \omega_2 = r_1 / r_2$$

If there is any sliding, that the contact is a way from the pitch point by a distance (e), then the sliding velocity (V_G) can be found as:

$$(V_G) = (\omega_1 + \omega_2)\, e$$

Interference:

Interference is a big disadvantage of the involute gear. It occurs when the tip of the tooth digs into the radial flank of the tooth in the pinion. Interference occurs when it desire to increase the addendum to the maximum possible, i.e. to increase the length of contact and hence to increase the number of teeth simultaneously in contact. The maximum possible addendum is when E, leis on F_2. If e_1 lies after f_2 interference occur. Normally interference is possible when the smallest gear meshes with largest gear, 12 tooth pinion and rack.

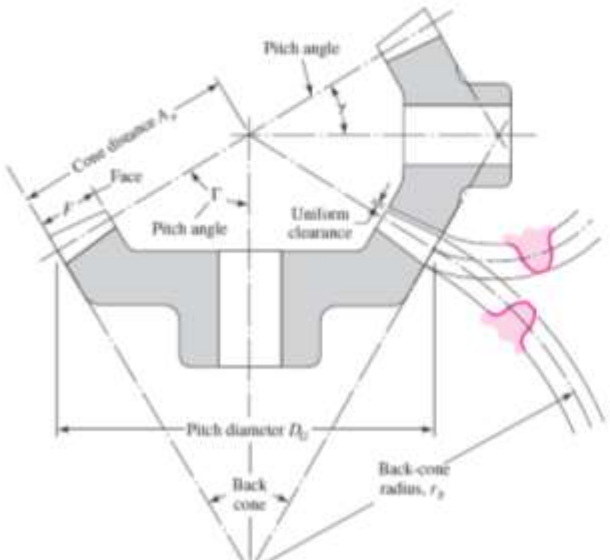


Figure : Terminology of bevel gears.

Table : Standard modules

Module (mm)	Equivalent P_d	Closest standard P_d (teeth/in)
0.3	84.667	64
0.4	63.500	48
0.5	50.800	32
0.8	31.750	24
1	25.400	20
1.25	20.320	16
1.5	16.933	12
2.5	12.700	10
2	10.160	8
3	8.466	6
4	6.350	5
5	5.080	4
6	4.233	3
8	3.175	2.5
10	2.540	2
12	2.117	1.5
16	1.587	1.25
20	1.270	1
25	1.016	80

Stresses in Spur and Helical Gears

This section is devoted primarily to analysis and design of spur and helical gears to resist bending failure of the teeth as well as pitting failure of tooth surfaces. Failure by bending will occur when the significant tooth stress equals or exceeds either the yield strength or the bending endurance strength. A surface failure occurs when the significant contact stress equals or exceeds the surface endurance strength.

The American Gear Manufacturers Association1 (AGMA) has for many years been the responsible authority for the dissemination of knowledge pertaining to the design and analysis of gearing. The general AGMA approach requires a great many charts and graphs—too many for shown here. We have omitted many of these here by choosing a single pressure angle and by using only full-depth teeth. This simplification reduces the complexity but does not prevent the development of a basic understanding of the approach. Furthermore, the simplification makes possible a better development of the fundamentals and hence should constitute an ideal introduction to the use of the general

The Lewis Bending Equation, AGMA Stress and Strength Equations

Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth in which the tooth form entered into the formulation. The equation, announced in 1892, still remains the basis for most gear design today.

To derive the basic Lewis equation, refer to following figure part *a*, which shows a cantilever of cross-sectional dimensions F and t, having a length l and a load W_t, uniformly distributed across the face width F (b in SI equations).

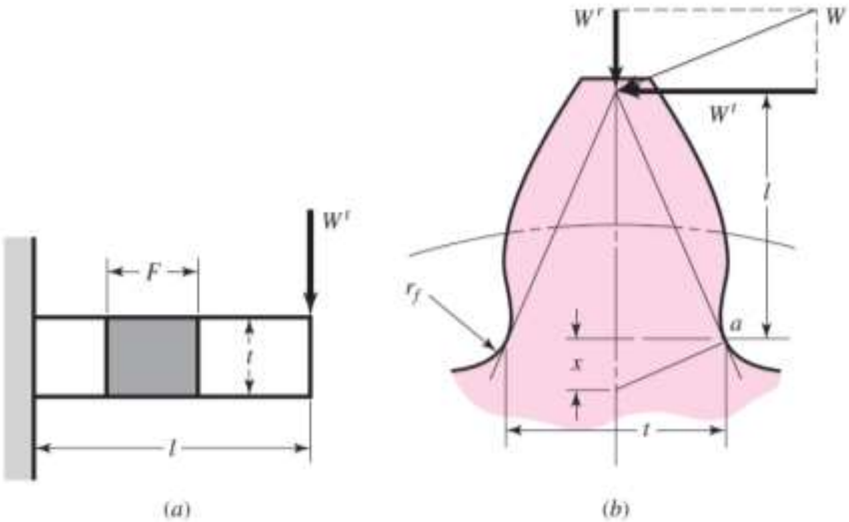


Figure G-1: a cantilever with uniformly distributed force *F* model of gear tooth

The derivation will not be presented in this introductory notes (for more detail see one of the design text books and please be aware of different symbols used in different text books) and the final metric equation will be presented.

Two fundamental stress equations are used in the AGMA methodology, one for bending stress and another for pitting resistance (contact stress). In AGMA terminology, these are called *stress numbers*, as contrasted with actual applied stresses, and are designated by a lowercase letter s instead of the Greek lower case σ we have used in this notes (and shall continue to use).

Bending Stress and strength

The fundamental equations are

$$\sigma = W^t K_v K_o K_s \frac{1}{b\, m} \frac{K_H K_B}{Y_J}$$

where for (SI units),

W^t is the tangential transmitted load, (N)
K_v is the dynamic factor
K_o is the overload factor
K_s is the size factor
b is the face width of the narrower member, in (mm)
K_H is the load-distribution factor
K_B is the rim-thickness factor
Y_J is the geometry factor for bending strength (which includes root fillet stress-concentration factor K_f)
m_t is the transverse metric module

Note here, that if the face width **b** and the module **m** are both in millimeters (**mm**). Expressing the tangential component of load W_t in Newton (N) then results in stress units of MegaPascals (MPa).

The equation for the allowable bending stress (SI units) is

$$\sigma_{all} = \frac{S_t}{S_F} \frac{Y_N}{Y_Z Y_\theta} \quad (SI\ units)$$

where

S_t is the allowable bending stress, (N/mm^2)

Y_N is the stress cycle factor for bending stress
Y_θ is the temperature factor
Y_Z is the reliability factor
S_F is the AGMA factor of safety, a stress ratio

Note here, instead of using the term *strength,* AGMA uses data termed *allowable stress numbers* and designates these by the symbols s_{at} and s_{ac}. It will be less confusing here if we continue the practice in these notes of using the uppercase letter *S* to designate strength and the lowercase Greek letters σ and τ for stress. To make it perfectly clear we shall use the term *gear strength* as a replacement for the phrase *allowable stress numbers* as used by AGMA. Following this convention, values for *gear bending strength,* designated here as S_t , are to be found in the following figures and tables. Since gear strengths are not identified with other strengths such as S_{ut} , S_e, or S_y as used elsewhere in these notes, their use should be restricted to gear problems. In this approach the strengths are modified by various factors that produce limiting values of the bending stress and the contact stress.

Contact Stress and strength (Pitting Resistance)
The fundamental equation for pitting resistance (contact stress) is

$$\sigma_c = Z_E \sqrt{W^t K_v K_o K_s \frac{K_H}{b\, d_{w1}} \frac{Z_R}{Z_I}} \quad (SI\ units)$$

where W^t, K_o, K_v , K_s, and *b* are the same terms as defined for bending equation. For SI units, the additional terms are
Z_E is an elastic coefficient, (N/mm2)
Z_R is the surface condition factor
d_{w1} is the pitch diameter of the *pinion,* (mm)
Z_I is the geometry factor for pitting resistance
The equation for the allowable contact stress $\sigma_{c,\text{all}}$ is

$$\sigma_{c,\ all} = \frac{S_c}{S_H} \frac{Z_N Z_W}{Y_Z Y_\theta} \quad (SI\ units)$$

where
S_c is the allowable contact stress, (N/mm^2)
Z_N is the stress cycle life factor
Z_W is are the hardness ratio factor for pitting resistance
Y_θ is the temperature factor
Y_Z is the reliability factor
S_H is the AGMA factor of safety, a stress ratio
The values for the allowable contact stress, designated here as S_c, are to be found in the following figures and Tables. AGMA allowable stress numbers (strengths) for bending and contact stress are for
- Unidirectional loading
- 10 million stress cycles
- 99 percent reliability

Before you try to digest the meaning of all these terms, view them as advice concerning items the designer should consider *whether he or she follows the voluntary standard or not*. These items include issues such as
- Transmitted load magnitude
- Overload

• Dynamic augmentation of transmitted load
• Size
• Geometry: pitch and face width
• Distribution of load across the teeth
• Rim support of the tooth
• Lewis form factor and root fillet stress concentration

Bending Stress And The Contact strengths:
In this approach the strengths are modified by various factors that produce limiting values of the bending stress and the contact stress. *(Source: ANSI/AGMA 2001-D04, 2101-D04.)*
*Allowable bending stress number for through-hardened steels. The SI equations are

$$St = 0.533HB + 88.3 \text{ MPa}, \quad \text{grade 1,}$$
$$St = 0.703HB + 113 \text{ MPa}, \quad \text{grade 2.}$$

*Allowable bending stress number for nitrided through-hardened steel gears (i.e., AISI 4140, 4340), *St* . The SI equations are

$$S_t = 0.568HB + 83.8 \text{ MPa, grade 1,}$$
$$S_t = 0.749HB + 110 \text{ MPa, grade 2.}$$

*Allowable bending stress numbers for nitriding steel gears S_t . The SI equations are

$$S_t = 0.594HB + 87.76 \text{ MPa} \quad \text{for Nitralloy grade 1}$$
$$S_t = 0.784HB + 114.81 \text{ MPa} \quad \text{for Nitralloy grade 2}$$
$$S_t = 0.7255HB + 63.89 \text{ MPa} \quad \text{for 2.5\% chrome, grade 1}$$
$$S_t = 0.7255HB + 153.63 \text{ MPa} \quad \text{for 2.5\% chrome, grade 2}$$
$$S_t = 0.7255HB + 201.91 \text{ MPa} \quad \text{for 2.5\% chrome, grade 3}$$

* Contact-fatigue strength S_c at 10^7 cycles and 0.99 reliability for through-hardened steel gears. The SI equations are

$$S_c = 2.22HB + 200 \text{ MPa}, \quad \text{grade 1}$$
$$S_c = 2.41HB + 237 \text{ MPa}, \quad \text{grade 2.}$$

*When two-way (reversed) loading occurs, as with idler gears, AGMA recommends using 70 percent of S_t values.

Lewis form factor (*Y*):
Values of *Y* are tabulated in Table G 1.

Table G-1: Values of the Lewis Form Factor *Y* (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	*Y*	Number of Teeth	*Y*
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Dynamic Effects (K_v)

When a pair of gears is driven at moderate or high speed and noise is generated, it is certain that dynamic effects are present. Note that the definition of dynamic factor K_v has been altered. AGMA standards. Dynamic factor K_v has been redefined as the reciprocal of that used in previous AGMA standards. It is now greater than 1.0. In earlier AGMA standards it was less than 1.0. Care must be taken in referring to work done prior to this change in the standards.
In SI units, we use the following equations.

$$K_v = \frac{3.05 + V}{3.05} \quad (cast\ iron, cast\ profile)$$

$$K_v = \frac{6.1 + V}{6.1} \quad (cut\ or\ milled\ profile)$$

$$K_v = \frac{3.56 + \sqrt{V}}{3.56} \quad (hobbed\ or\ shaped\ profile)$$

$$K_v = \sqrt{\frac{5.56 + \sqrt{V}}{5.56}} \quad (shaved\ or\ ground\ profile)$$

where *V* is in meters per second (m/s).

Geometry Factors *(Z_I)* and *(Y_J)*

We have seen how the factor *Y* is used in the Lewis equation to introduce the effect of tooth form into the stress equation. The AGMA factors Z_I and Y_J are intended to accomplish the same purpose in a more involved manner.
The determination of Z_I and Y_J depends upon the *face-contact ratio* m_b . This is defined as

$$\boldsymbol{m_b = b/p_x}$$

where p_x is the axial pitch and *b* is the face width. For spur gears, $m_b = 0$.
Low-contact-ratio (LCR) helical gears having a small helix angle or a thin face width, or both, have face-contact ratios less than unity ($m_b \le 1$), and will not be considered here. Such gears have a noise level not too different from that for spur gears.

Consequently we shall consider here only spur gears with m_b = 0 and conventional helical gears with *mb* > 1.
The load-sharing ratio m_N is equal to the face width divided by the minimum total length of the lines of contact. This factor depends on the transverse contact ratio m_p, the face-contact ratio m_b , the effects of any profile modifications, and the tooth deflection. For spur gears, m_N = 1.0. For helical gears having a face-contact ratio *mF* > 2.0, a conservative approximation is given by the equation

$$m_N = p_N/0.95Z$$

where p_N is the normal base pitch and *Z* is the length of the line of action in the transverse plane Use the following figure to obtain the geometry factor Y_J for spur gears having a 20° pressure angle and full-depth teeth. Use the next figures for helical gears having a 20° normal pressure angle and face-contact ratios of m_b = 2 or greater. For other gears, consult the AGMA standard.

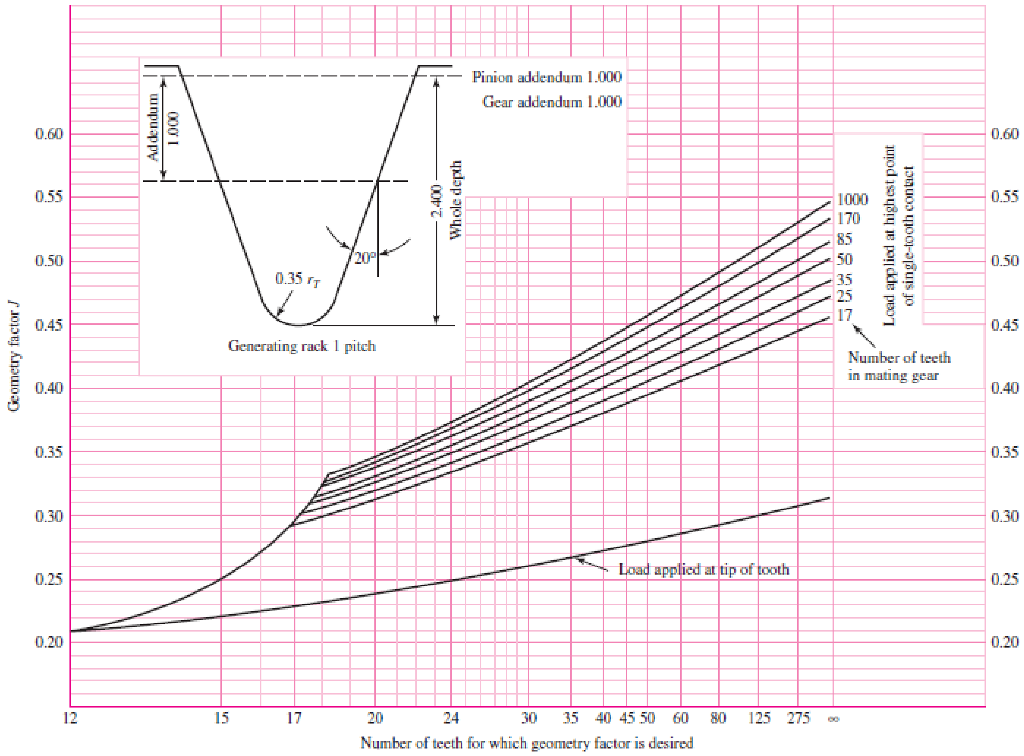
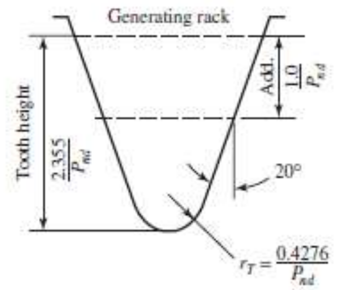
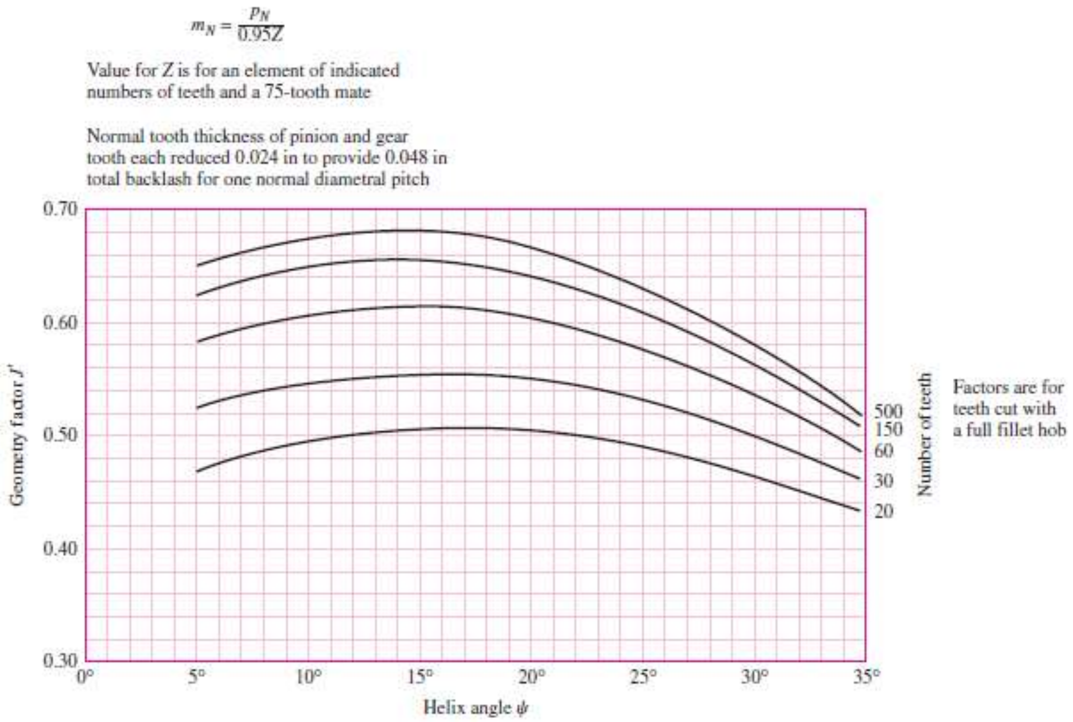


Figure Spur-gear geometry factors Y_J. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



(a)



(b)

Figure Helical-gear geometry factors Y_J _. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes

The modifying factor can be applied to the Y_J factor when other than 75 teeth are used in the mating element

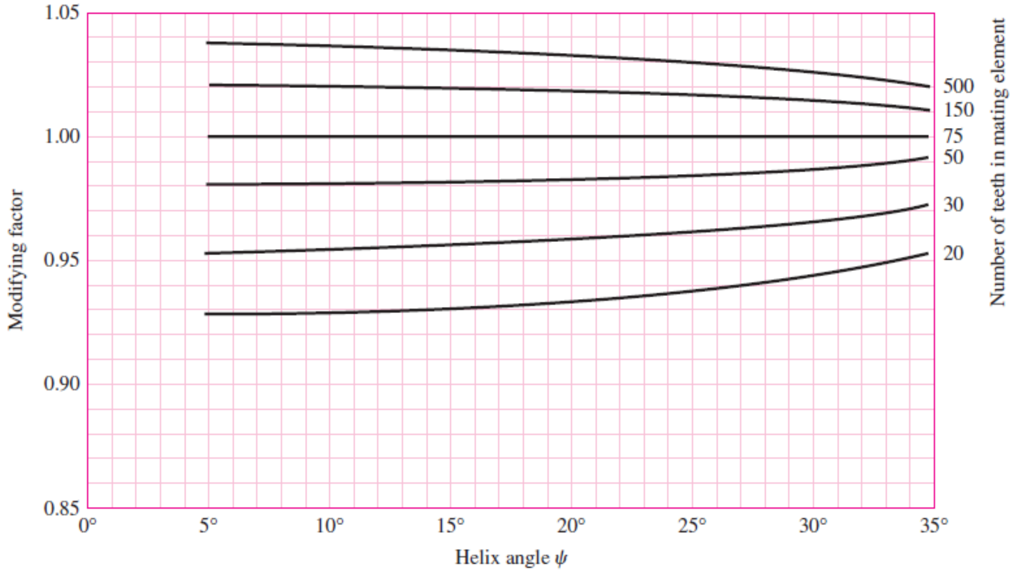


Figure Y_J '-factor multipliers for use with the previous figure to find Y_J. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes

Surface-Strength Geometry Factor (Z_I)

The factor Z_I is also called the *pitting-resistance geometry factor* by AGMA. We will develop an expression for Z_I.

Now define *speed ratio* m_G as

$$\boldsymbol{m_G = N_G/N_P = d_G/d_P}$$

The geometry factor Z_I for external spur and helical gears by adding the load-sharing ratio m_N , we obtain a factor valid for both spur and helical gears. The equation is then written as

$$Z_I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G+1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G-1} & \text{internal gears} \end{cases}$$

where $m_N = 1$ for spur gears.

Certain precautions must be taken in using the above. The tooth profiles are not conjugate below the base circle, and consequently, if either one or the other of the first two terms in brackets is larger than the third term, then it should be replaced by the third term. In addition, the effective outside radius is sometimes less than $r + a$, owing to removal of burrs or rounding of the tips of the teeth. When this is the case, always use the effective outside radius instead of $r + a$.

The Elastic Coefficient (Z_E)

Values of Z_E may be computed directly from the following equation or obtained from Table.

$$Z_E = \left[\frac{1}{\pi\left(\dfrac{1-\nu_P^2}{E_P} + \dfrac{1-\nu_G^2}{E_G}\right)}\right]^{1/2}$$

Table G-2 Elastic Coefficient C_p (Z_E,), √psi (√MPa) *Source: AGMA 218.01*

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in^2 (MPa)*					
		Steel	Malleable Iron	Nodular Iron	Cast Iron	Aluminum Bronze	Tin Bronze
		30×10^6	25×10^6	24×10^6	22×10^6	17.5×10^6	16×10^6
		(2×10^5)	(1.7×10^5)	(1.7×10^5)	(1.5×10^5)	(1.2×10^5)	(1.1×10^5)
Steel	30×10^6	2300	2180	2160	2100	1950	1900
	(2×10^5)	(191)	(181)	(179)	(174)	(162)	(158)
Malleable iron	25×10^6	2180	2090	2070	2020	1900	1850
	($1.7\times10_5$)	(181)	(174)	(172)	(168)	(158)	(154)
Nodular iron	24×10^6	2160	2070	2050	2000	1880	1830
	(1.7×10^5)	(179)	(172)	(170)	(166)	(156)	(152)
Cast iron	22×10^6	2100	2020	2000	1960	1850	1800
	(1.5×10^5)	(174)	(168)	(166)	(163)	(154)	(149)
Aluminum bronze	17.5×10^6	1950	1900	1880	1850	1750	1700
	(1.2×10^5)	(162)	(158)	(156)	(154)	(145)	(141)
Tin bronze	16×10^6	1900	1850	1830	1800	1700	1650
	($1.1\times10_5$)	(158)	(154)	(152)	(149)	(141)	(137)

Poisson's ratio = 0.30.
∗When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

Overload Factor *(K_o)*

The overload factor K_o is intended to make allowance for all externally applied loads in excess of the nominal tangential load W_t in a particular application. Examples include variations in torque from the mean value due to firing of cylinders in an internal combustion engine or reaction to torque variations in a piston pump drive. There are other similar factors such as application factor or service factor. These factors are established after considerable field experience in a particular application.
The role of the overload factor K_o is to include predictable excursions of load beyond W^t based on experience. A safety factor is intended to account for unquantifiable elements in addition to K_o. When designing a gear mesh, the quantity S_F becomes a design factor $(S_F)_d$ within the meanings used in these notes. The quantity S_F evaluated as part of a design assessment is a factor of safety. This applies equally well to the quantity S_H .

Table: of Overload Factors, K_o

	Driven Machine		
Power source	**Uniform**	**Light shock**	**Medium shock**
Uniform	1.00	1.25	1.50
Moderate shock	1.25	1.50	1.75
Heavy shock	1.75	2.00	2.25

Surface Condition Factor (Z_R)

The surface condition factor Z_R is used only in the pitting resistance equation. It depends on

- Surface finish as affected by, but not limited to, cutting, shaving, lapping, grinding, shot-peening
- Residual stress

- Plastic effects (work hardening)

Standard surface conditions for gear teeth have not yet been established. When a detrimental surface finish effect is known to exist, AGMA specifies a value of Z_R greater than unity.

Size Factor *(K_s)*

The size factor reflects non-uniformity of material properties due to size. It depends upon

- Tooth size
- Diameter of part
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment

Standard size factors for gear teeth have not yet been established for cases where there is a detrimental size effect. In such cases AGMA recommends a size factor greater than unity. If there is no detrimental size effect, use unity. AGMA has identified and provided a symbol for size factor. Also, AGMA suggests K_s = 1, which makes K_s a placeholder until more information is gathered. Following the standard in this manner is a failure to apply all of your knowledge.

Noting that K_s is the reciprocal of k_b , we find the result of all the algebraic substitution is

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535}$$

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{b\sqrt{Y}}{P}\right)^{0.0535}$$

K_s can be viewed as Lewis's geometry incorporated into the Marin size factor in fatigue. You may set K_s = 1, or you may elect to use the preceding equation. This is a point to discuss with your instructor. We will use the preceding equation to remind you that you have a choice. If K_s in the equatiois less than 1, use K_s = 1.

Load-Distribution Factor (K_H)

The load-distribution factor modified the stress equations to reflect nonuniform distribution of load across the line of contact. The ideal is to locate the gear "midspan" between two bearings at the zero slope place when the load is applied. However, this is not always possible. The following procedure is applicable to

- Net face width to pinion pitch diameter ratio $F/d \leq 2$
- Gear elements mounted between the bearings
- Face widths up to 40 in
- Contact, when loaded, across the full width of the narrowest member

The load-distribution factor under these conditions is currently given by the *face load distribution factor, C_{mf}* , where

$$K_H = C_{mf} = 1 + C_{mc}(C_{pf}\, C_{pm} + C_{ma}C_e)$$

Hardness-Ratio Factor C_H

The pinion generally has a smaller number of teeth than the gear and consequently is subjected to more cycles of contact stress. If both the pinion and the gear are through-hardened, then a uniform surface strength can be obtained by making the pinion harder than the gear. A similar effect can be obtained when a surface-hardened pinion is mated with a through hardened gear. The hardness-ratio factor C_H is used *only for the gear*. Its purpose is to adjust the surface strengths for this effect. The values of C_H are obtained from the equation

$$C_H = 1.0 + A'(m_G - 1.0)$$

where

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3}) \quad 1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7$$

The terms H_{BP} and H_{BG} are the Brinell hardness (10-mm ball at 3000-kg load) of the pinion and gear, respectively. The term m_G is the speed ratio and is given before.

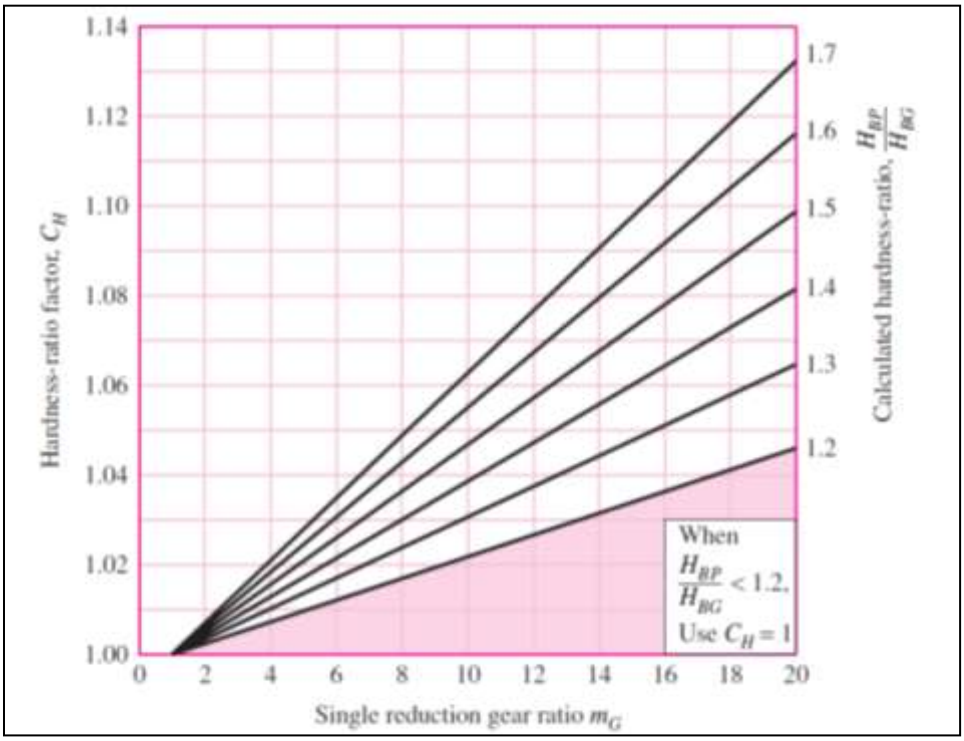


Figure: Hardness-ratio factor C_H(through-hardened steel). *(ANSI/AGMA 2001-D04.)*

For

$$\frac{H_{BP}}{H_{BG}} < 1.2, \quad A' = 0$$

$$\frac{H_{BP}}{H_{BG}} > 1.7, \quad A' = 0.006\,98$$

When surface-hardened pinions with hardnesses of 48 Rockwell C scale (Rockwell C48) or harder are run with through-hardened gears (180–400 Brinell), a work hardening occurs. The C_H factor is a function of pinion surface finish f_P and the mating gear hardness. The following figure displays the relationships.

$$\boldsymbol{C_H = 1 + B'\,(450 - H_{BG})}$$

where $B' = 0.000\,75 \exp[-0.0112\, f_P]$ and f_P is the surface finish of the pinion expressed as root-mean-square roughness R_a in μ in.

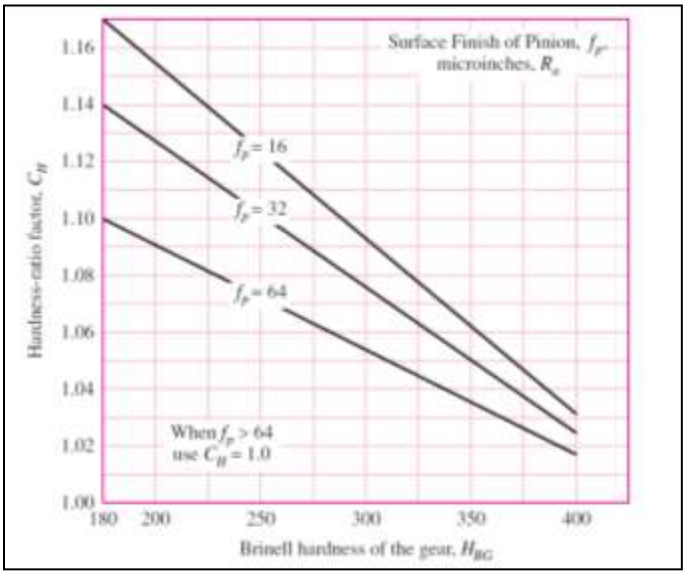


Figure : Hardness-ratio factor C_H (surface-hardened steel pinion). (ANSI/AGMA 2001-D04.)

Stress-Cycle Factors *(Y_N)* and *(Z_N)*

The AGMA strengths as given before in figures and in Tables for bending fatigue, and for contact-stress fatigue are based on 10^7 load cycles applied. The purpose of the load cycle factors Y_N and Z_N *are* to modify the gear strength for lives other than 10^7 cycles. Values for these factors are given in the following figures. Note that for 10^7 cycles $Y_N = Z_N = 1$ on each graph. Note also that the equations for Y_N and Z_N change on either side of 10^7 cycles. For life goals slightly higher than 10^7 cycles, the mating gear may be experiencing fewer than 10^7 cycles and the equations for $(Y_N)_P$ and $(Y_N)_G$ can be different. The same comment applies to $(Z_N)_P$ and $(Z_N)_G$.

Table : Recommended design life

Application	**Design life (h)**
Domestic appliances	1000-2000
Aircraft engines	1000-4000
Automotive	1500-5000
Agricultural equipment	3000-6000
Elevators, industrial fans, multipurpose gearing	8000-15 000
Electric motors, industrial blowers, general industrial machines	20 000-30 000
Pumps and compressors	40 000-60 000
Critical equipment in continuous 24-h operation	100 000-200 000

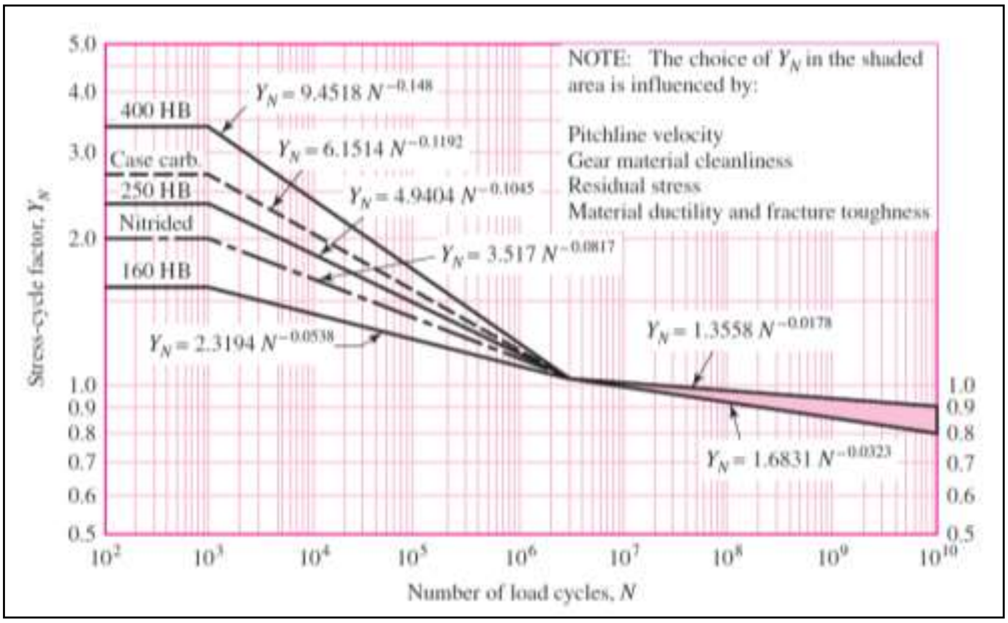


Figure : Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*

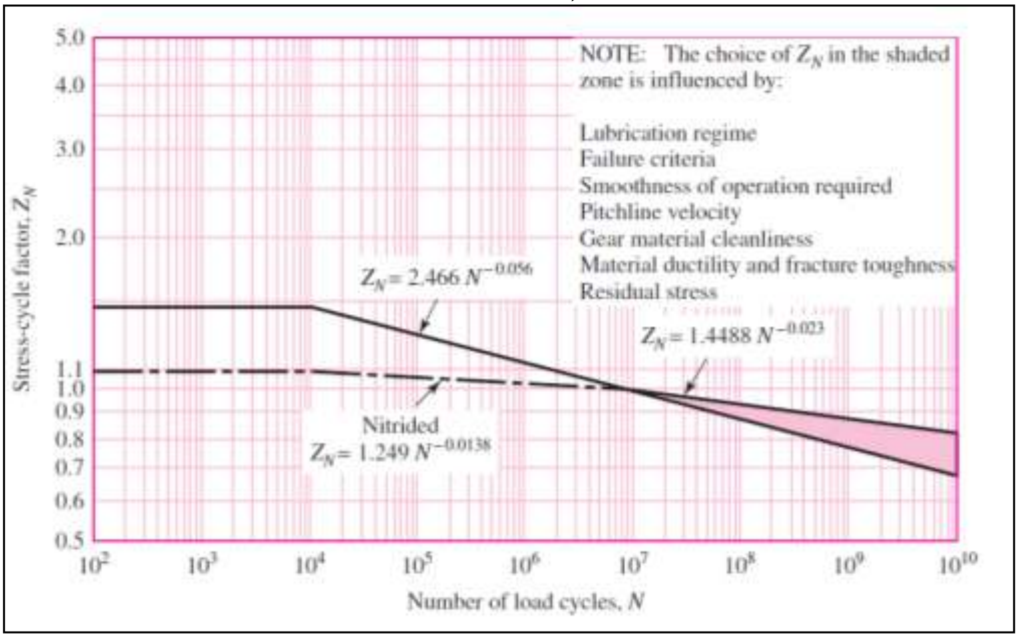


Figure: Pitting resistance stress-cycle factor Z_N. *(ANSI/AGMA 2001-D04.)*

Reliability Factor (Y_Z)

The reliability factor accounts for the effect of the statistical distributions of material fatigue failures. Load variation is not addressed here. The gear strengths S_t and S_c are based on a reliability of 99 percent. The following table is based on data developed by the U.S. Navy for bending and contact-stress fatigue failures. The functional relationship between K_R and reliability is highly nonlinear. When interpolation is required, linear interpolation is too crude. A log transformation to each quantity produces a linear string. A least-squares regression fit is

$$Y_Z = \begin{Bmatrix} 0.658 - 0.0759\ln(1-R) & 0.5 < R < 0.99 \\ 0.5 - 0.109\ln(1-R) & 0.99 < R < 0.9999 \end{Bmatrix}$$

For cardinal values of R, take K_R from the table. Otherwise use the logarithmic interpolation afforded.

Table G-3 Reliability Factors *(Y_Z) Source: ANSI/AGMA 2001-D04.*

Reliability	(YZ)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

Temperature Factor (Y_θ)

For oil or gear-blank temperatures up to 250°F (120°C), use $K_T = Y_\theta = 1.0$. For higher temperatures, the factor should be greater than unity. Heat exchangers may be used to ensure that operating temperatures are considerably below this value, as is desirable for the lubricant.

Rim-Thickness Factor *(K_B)*

When the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim rather than at the tooth fillet. In such cases, the use of a stress-modifying factor K_B or (t_R) is recommended. This factor, the *rim-thickness factor* K_B, adjusts the estimated bending stress for the thin-rimmed gear. It is a function of the backup ratio m_B,

$$m_B = t_R / h_t$$

where t_R = rim thickness below the tooth, in, and h_t = the tooth height. The geometry is depicted in the following figure. The rim-thickness factor K_B is given by

$$Y_Z = \begin{Bmatrix} 1.6\ \ln\dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{Bmatrix}$$

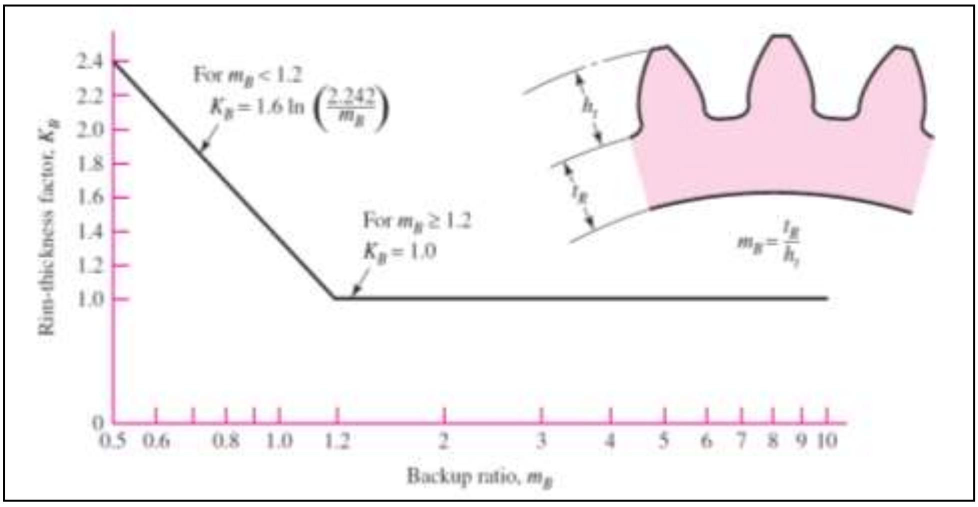


Figure : Rim-thickness factor K_B. *(ANSI/AGMA 2001-D04.)*

Safety Factors (S_F*) and (*S_H*)*

The ANSI/AGMA standards 2001-D04 and 2101-D04 contain a safety factor S_F guarding against bending fatigue failure and safety factor S_H guarding against pitting failure.

The definition of S_F is,

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{Fullu\ corrected\ contact\ strength}{contact\ stress}$$

when σ_c is estimated as before. This, too, is a strength-over-stress definition but in a case where the stress is *not* linear with the transmitted load W^t .

While the definition of S_H does not interfere with its intended function, a caution is required when comparing S_F with S_H in an analysis in order to ascertain the nature and severity of the threat to loss of function. To render S_H linear with the transmitted load, W^t it could have been defined as

$$S_H = \left(\frac{Fullu\ corrected\ contact\ strength}{contact\ stress}\right)^2$$

with the exponent 2 for linear or helical contact, or an exponent of 3 for crowned teeth (spherical contact). With the definition compare S_F with S_{2H} (or S_{3H} for crowned teeth) when trying to identify the threat to loss of function with confidence.

Design of a Gear Mesh

A useful decision set for spur and helical gears includes

a priori decisions:

- Function: load, speed, reliability, life, K_o
- Unquantifiable risk: design factor n_d
- Tooth system: φ, ψ, addendum, dedendum, root fillet radius
- Gear ratio m_G, N_p, N_G
- Quality number Q_v

design decisions :

- Module m
- Face width b
- Pinion material, core hardness, case hardness
- Gear material, core hardness, case hardness

The first item to notice is the dimensionality of the decision set. There are four design decision categories, eight different decisions if you count them separately. This is a larger number than we have encountered before. It is important to use a design strategy that is convenient in either longhand execution or computer implementation. The design decisions have been placed in order of importance (impact on the amount of work to be redone in iterations). The steps, after the a priori decisions have been made are:

- Choose a Model.
- Examine implications on face width, pitch diameters, and material properties. If not satisfactory, return to pitch decision for change.
- Choose a pinion material and examine core and case hardness requirements. If not satisfactory, return to pitch decision and iterate until no decisions are changed.
- Choose a gear material and examine core and case hardness requirements. If not satisfactory, return to pitch decision and iterate until no decisions are changed.

With these plan steps in mind, we can consider them in more detail. First select a trial Module pitch.

Pinion bending:

- Select a median face width for this pitch, $4\pi/P$
- Find the range of necessary ultimate strengths
- Choose a material and a core hardness
- Find face width to meet factor of safety in bending
- Choose face width
- Check factor of safety in bending

Gear bending:

- Find necessary companion core hardness
- Choose a material and core hardness
- Check factor of safety in bending

Pinion wear:

- Find necessary *Sc* and attendant case hardness
- Choose a case hardness
- Check factor of safety in wear

Gear wear:

- Find companion case hardness
- Choose a case hardness
- Check factor of safety in wear

Completing this set of steps will yield a satisfactory design. Additional designs with Module pitches adjacent to the first satisfactory design will produce several among which to choose. A figure of merit is necessary in order to choose the best. Unfortunately, a figure of merit in gear design is complex in an academic environment because material and processing cost vary. The possibility of using a process depends on the manufacturing facility if gears are made in house.

After examining and seeing the wide range of factors of safety, one might entertain the notion of setting all factors of safety equal. In steel gears, wear is usually controlling and $(S_H)_P$ and $(S_H)_G$ can be brought close to equality. The use of softer cores can bring down $(S_F)_P$ and $(S_F)_G$, but there is value in keeping them higher. A tooth broken by bending fatigue not only can destroy the gear set, but can bend shafts, damage bearings, and produce inertial stresses up- and downstream in the power train, causing damage elsewhere if the gear box locks.

To have a satisfactory design for mesh. Material could be changed, as could pitch. There are a number of other satisfactory designs, thus a figure of merit is needed to identify the best. One can appreciate that gear design was one of the early applications of the digital computer to mechanical engineering. A design program should be interactive, presenting results of calculations, pausing for a decision by the designer, and showing the consequences of the decision, with a loop back to change a decision for the better. The program can be structured in totem-pole fashion, with the most influential decision at the top, then tumbling down, decision after decision, ending with the ability to change the current decision or to begin again. Such a program would make a fine class project. Troubleshooting the coding will reinforce your knowledge, adding flexibility as well as bells and whistles in subsequent terms. Standard gears may not be the most economical design that meets the functional requirements, because no application is standard in all respects.10 Methods of designing custom gears are well understood and frequently used in mobile equipment to provide good weight-to-performance index. The required calculations including optimizations are within the capability of a personal computer.

Bevel and Worm Gears

The American Gear Manufacturers Association (AGMA) has established standards for the analysis and design of the various kinds of bevel and worm gears. The previous sections are an introduction to the AGMA methods for spur and helical gears. AGMA has established similar methods for other types of gearing, which all follow the same general approach.

Bevel Gearing—General

Bevel gears may be classified as follows:

- Straight bevel gears
- Spiral bevel gears
- Zerol bevel gears
- Hypoid gears
- Spiroid gears

A straight bevel gears are usually used for pitch-line velocities up to 1000 ft/min (5 m/s) when the noise level is not an important consideration. They are available in many stock sizes and are less expensive to produce than other bevel gears, especially in small quantities.

A spiral bevel gears are recommended for higher speeds and where the noise level is an important consideration. Spiral bevel gears are the bevel counterpart of the helical gear; it can be seen that the pitch surfaces and the nature of contact are the same as for straight bevel gears except for the differences brought about by the spiral-shaped teeth.

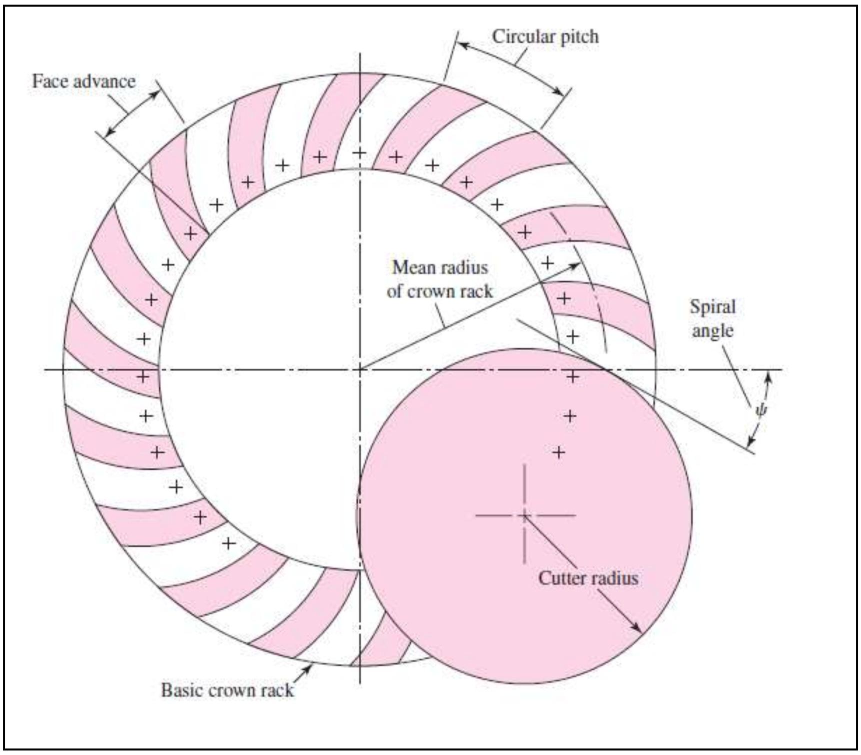


Figure : Cutting spiral-gear teeth on the basic crown rack.

The ***Zerol bevel gear*** is a patented gear having curved teeth but with a zero spiral angle. The axial thrust loads permissible for Zerol bevel gears are not as large as those for the spiral bevel gear, and so they are often used instead of straight bevel gears. The Zerol bevel gear is generated by the same tool used for regular spiral bevel gears. For design purposes, use the same procedure as for straight bevel gears and then simply substitute a Zerol bevel gear.

It is frequently desirable, as in the case of automotive differential applications, to have gearing similar to bevel gears but with the shafts offset. Such gears are called ***hypoid***

gears, because their pitch surfaces are hyperboloids of revolution. The tooth action between such gears is a combination of rolling and sliding along a straight line and has much in common with that of worm gears. Figure shows a pair of hypoid gears in mesh. Figure is included to assist in the classification of spiral bevel gearing. It is seen that the hypoid gear has a relatively small shaft offset. For larger offsets, the pinion begins to resemble a tapered worm and the set is then called ***spiroid gearing***.

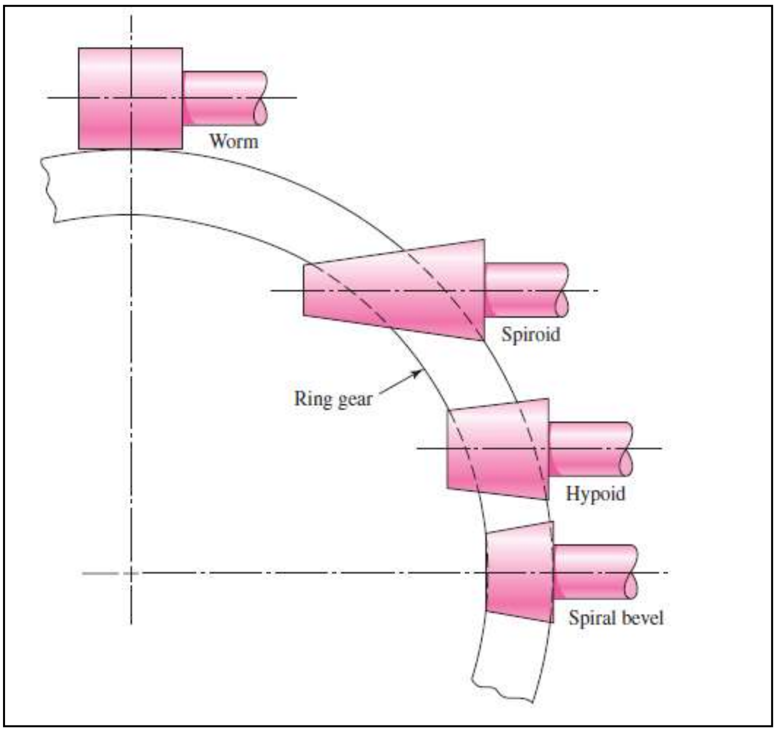


Figure : Comparison of intersecting and offset-shaft bevel-type gearings. *(From* Gear Handbook *by Darle W. Dudley, 1962, pp. 2–24.)*

Bevel-Gear Stresses and Strengths

In a typical bevel-gear mounting, one of the gears is often mounted outboard of the bearings. This means that the shaft deflections can be more pronounced and can have a greater effect on the nature of the tooth contact. Another difficulty that occurs in predicting the stress in bevel-gear teeth is the fact that the teeth are tapered. Thus, to achieve perfect line contact passing through the cone center, the teeth ought to bend more at the large end than at the small end. To obtain this condition requires that the load be proportionately greater at the large end. Because of this varying load across the face of the tooth, it is desirable to have a fairly short face width. Because of the complexity of bevel, spiral bevel, Zerol bevel, hypoid, and spiroid gears, as well as the limitations of space, only an introduction that refer to straight-bevel gears is presented here. Straight-bevel gears are designed in similar method as spur and helical gears with a small deviations in dealing with factors and introducing new factors as will. Details of design will be left for student to learn if needed.

Worm Gearing—AGMA Equation

Since they are essentially non-enveloping worm gears, the *crossed helical* gears, shown in the following figure, can be considered with other worm gearing.

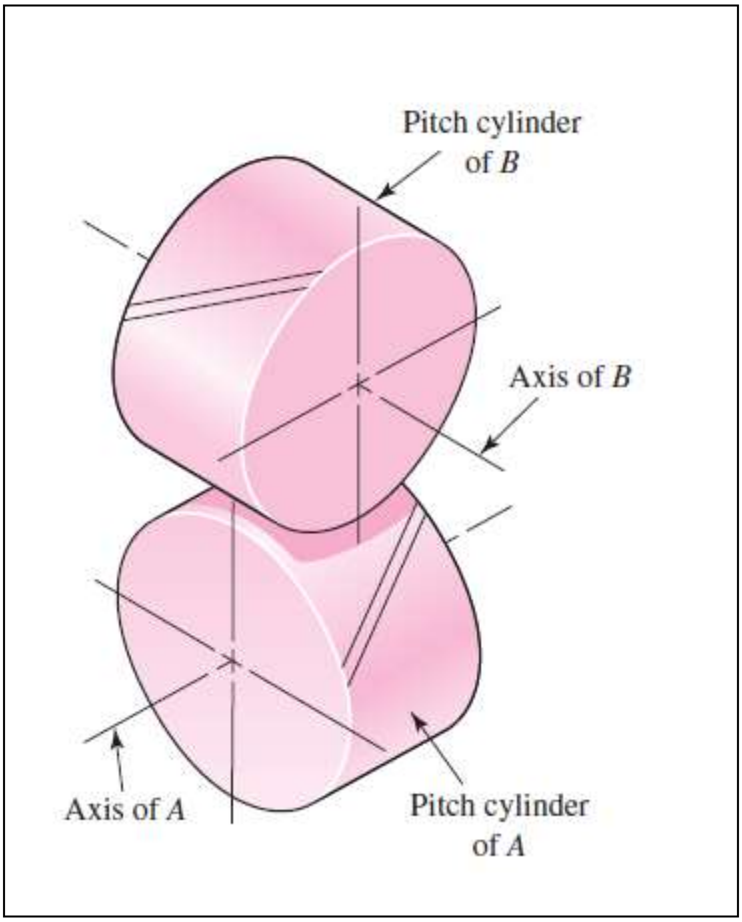


Figure : View of the pitch cylinders of a pair of crossed helical gears.

Because the teeth of worm gears have *point contact* changing to *line contact* as the gears are used, worm gears are said to "wear in," whereas other types "wear out." Crossed helical gears, and worm gears too, usually have a 90° shaft angle, though this need not be so. The relation between the shaft and helix angles is

$$\sum = \psi_P \pm \psi_G$$

where $\sum$ is the shaft angle. The plus sign is used when both helix angles are of the same hand, and the minus sign when they are of opposite hand. The subscript $_P$ refers to the pinion (worm); the subscript $_W$ is used for this same purpose. The subscript $_G$ refers to the gear, also called *gear wheel, worm wheel,* or simply the *wheel*.

In the force calculation section we introduced worm gears, and developed the force analysis and efficiency of worm gearing to which we will refer. Here our interest is in strength and durability. Good proportions indicate the pitch worm diameter *d* falls in the range

$$\frac{C^{0.875}}{3} \le d \le \frac{C^{0.875}}{1.6}$$

where *C* is the center-to-center distance. AGMA relates the allowable tangential force on the worm-gear tooth *(W^t)* all to other parameters.

Compared to other gearing systems worm-gear meshes have a much lower mechanical efficiency. Cooling, for the benefit of the lubricant, becomes a design constraint sometimes resulting in what appears to be an oversize gear case in light of its contents. If the heat can be dissipated by natural cooling, or simply with a fan on

the worm-shaft, simplicity persists. Water coils within the gear case or lubricant outputmping to an external cooler is the next level of complexity. For this reason, gear-case area is a design decision. To reduce cooling load, use multiple-thread worms. Also keep the worm pitch diameter as small as possible. Multiple-thread worms can remove the self-locking feature of many worm-gear drives.
We will stop discussion here, and details of design will be left, too, for student to learn if needed.